Throwing Darts and Missing the Bullseye: Fiscal and Debt Sustainability in Open Economies*

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October 19, 2021

Abstract

This paper examines fiscal and debt sustainability in an open-economy, stock-flow consistent (SFC), and continuous-time model. It reproduces the stylised facts of pro-cyclical fiscal deficits and exchange rates, and counter-cyclical foreign assets held by the central bank. The model demonstrates that the government's budget constraint yields a fiscal rule that is not SFC, thus, it overshoots and undershoots the primary fiscal balance consistent with stock-flow equilibria (the bullseye). A stock-flow consistent fiscal rule shows that a primary fiscal deficit as a share of GDP obtains goods market equilibrium at potential output and a steady-state debt ratio, even when the economy is dynamically efficient and irrespective of the exchange rate regime. Moreover, the model finds that debt sustainability is compromised if pandemic budgets are money-financed. These results suggest that a long-run primary surplus is not consistent with debt sustainability, and a stock-flow inconsistent fiscal rule engenders economic cycles through foreign exchange and debt crises.

Keywords: debt sustainability, fiscal deficit, stock-flow equilibria, economic cycles
JEL Classification: E32, E62, F31, F41, H62, H63

*I thank Tridib Bhattacharya for excellent research assistance, and Tarron Khemraj for insightful discussions. I am also grateful to Giorgos Gouzoulis and Dillon Alleyne for helpful comments. The usual disclaimer applies. This paper is prepared for the 52nd Annual Monetary Studies Conference: Fiscal and Monetary Policy in the Caribbean in Pandemic Times, Trinidad & Tobago, 2021.

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Large primary surpluses have been frequent, but sustained large surpluses have been less common, [...] Out of a sample of 87 countries [...] only 16 countries (less than 20 percent) sustained primary surpluses exceeding 5 percent of GDP for five years or longer [...] episodes of sustained large surpluses were related to [...] natural resources [...] or transfers [...]

— IMF 2011: pp.8.

1 Introduction

What determines fiscal and public debt sustainability?[1] This question lurks in the background as governments respond in real-time to the ongoing pandemic. Presently, there is consensus that governments must do *whatever it takes* to save lives and restore the economy (Baldwin and Weder di Mauro 2020). But the recent debate on the inflationary and debt sustainability effects of the Biden budget (Stewart 2021), and growing concerns over the long-term debt scars in developing and emerging markets demonstrate the fragility of this consensus (Barret et al. 2021). A key factor in the present debate in the USA is the interest rate-growth rate inequality $\left(r - g \lesseqgtr 0\right)$. Based on the Domar debt sustainability condition, when the long-term rate of interest r is less (greater) than the long-run rate of economic growth g, only a primary fiscal deficit (surplus) can stabilise the public debt ratio (Domar 1944).[2] In smaller economies, the debate rests on the size of the public debt ratio and how it affects the interest rate-growth rate inequality (Lian et al. 2020; Greenidge et al. 2012; Reinhart and Rogoff 2010). These scholars argue that $r > g$ when the debt ratio is above a critical threshold, ergo, fiscal surpluses are necessary for debt sustainability.

In this paper, I demonstrate that the Domar debt sustainability condition is stock-flow inconsistent, and only by accident or a debt crisis does its fiscal rule obtain a steady-state debt ratio. The importance of this point should not be missed. It implies that a long-run primary surplus does not stabilise the public debt ratio when $r > g$, as stock-flow equilibria require the long-run accumulation of private sector debt and external surplus to ensure goods market equilibrium at potential output. But these produce a private sector debt crisis, deteriorate balance sheets, and necessarily reverse the long-run primary surplus for stabilisation purposes. Consistent with the evidence of

[1] In this paper public debt refers to the sum of domestic and foreign bonds issued by the central government, where the latter is denominated in foreign currency units.

[2] See Mauro and Zhou (2021) for empirical evidence in both advanced and developing economies that a *negative* interest rate-growth rate inequality has persisted for long periods, notwithstanding recurrent debt crises. Ball et al. (1998) and Blanchard (2019) present similar evidence for the case of the USA, and Escolano et al. (2017) for developing economies.

pro-cyclical fiscal deficits and exchange rates, and counter-cyclical foreign assets held by the central bank (Figure 1), the paper demonstrates that a debt-targeting fiscal rule is akin to *throwing darts and missing the bullseye*, where the latter is a primary fiscal balance that stabilises both debt stock and flow equilibrium. The stock-flow inconsistency emerges because the debt-targeting fiscal rule is derived from the government's budget identity or constraint. Except by coincidence, there is no guarantee the derived fiscal rule is also consistent with goods market (flow) equilibrium.

In this paper, I develop an open-economy, stock-flow consistent, and continuous-time model of fiscal and debt sustainability—this ensures that the primary fiscal balance as a share of GDP that stabilises the debt stock is also consistent with goods market equilibrium at potential output. The stock-flow model turns the Domar debt sustainability condition on its head. In the case of a fixed exchange rate regime, the paper shows that a bigger primary fiscal deficit accelerates the total public debt-GDP ratio, while foreign assets held by the private sector and the central bank, and the private sector's domestic debt as shares of GDP induce decelerations in the share of total public debt. Moreover, the latter explodes when the weighted rate of interest plus the share of government's external debt in total public debt exceed the long-run rate of growth, and the reverse is true. In the case of a pure float, the primary fiscal deficit and the private sector's domestic debt as shares of GDP accelerates and contracts the public debt ratio respectively. Further, the latter rises when the weighted rate of interest plus the long-run rate of currency depreciation exceed the long-run rate of growth, and the reverse holds.[3]

The intuition is as follows. As the central bank's stock of foreign assets as a share of GDP increases, it necessarily means that there is a trade surplus (or long-term inflows like FDI), which provides for falling public debt as the government becomes a net lender in world markets.[4] The same result holds if the private sector's stock of foreign assets as a share of GDP rises. Similarly, as the private sector's stock of domestic debt as a share of GDP expands so does domestic demand, and sectoral balance requires fewer holdings of public sector domestic debt. In other words, as the private sector increases its own domestic liabilities it is not simultaneously accumulating public sector debt. Finally, the public debt ratio accelerates as the stock of external debt in total public debt increases. Underlying this result is the transmission channel that more foreign assets are quarantined for servicing the external debt rather than accelerating long-run growth. In the case of a pure float, as the long-run rate of currency depreciation rises, the cost of purchasing foreign

[3] Canofari et al. (2020) recognise that debt analyses are inconsistent with stock-flow equilibria. Their debt condition is an improvement as it accounts for private wealth but it does not admit an external sector.

[4] Alternatively, higher reserves reduce the incentives of domestic savers to hold foreign currency assets for precautionary reasons, which lower the risk premium/interest rate on domestic debt (Bocola and Lorenzoni 2020).

currencies to service external debt obligations increases in local currency units, which accelerates the the local currency burden of public debt.

The key result from the model is that unless the positive weighted interest rate-growth rate inequality is *implausibly large*—the dynamic efficiency condition (Blanchard and Weil 2001)—a long-run primary fiscal deficit as a share of GDP stabilises the public debt ratio and the goods market at potential output in both exchange rate regimes. The driving force behind this result is the consideration of flow equilibrium.[5] Further, consistent with the evidence of low debt tolerance in developing countries and emerging markets (Reinhart et al. 2003), the model demonstrates that the stock-flow consistent debt sustainability conditions do have a shorter half-life as compared to the Domar condition. This implies that there is a tight upper limit on external debt and a faster speed of convergence to the long-run debt ratio, or indeed a debt crisis. The intuition relates to the fact that there is an upper limit on the stock of foreign assets central banks can hold in a pegged regime, and a sovereign's ability to borrow abroad in a pure float is constrained by its long-run rate of currency depreciation.

The stock-flow model leads to a new perspective on the concept of *fiscal space*. The latter refers to the *extent of resource availability* and *degree of resource utilisation* consistent with a stable debt ratio and goods market equilibrium at potential output. In a pegged exchange rate regime, resource availability depends on the central bank's stock of foreign assets as a share of GDP, and the long-run rate of currency depreciation in a pure float, where the latter determines the ability to borrow. In turn, the degree of resource utilisation depends on the private sector's stock of foreign assets and domestic liabilities as shares of GDP. Any sovereign that obeys this concept of fiscal space avoids debt/currency crises and realises more stable economic performance. It is not unlike Ghosh et al. (2013)'s formulation, where fiscal space is defined as the difference between the country's current debt level and its debt limit. The latter is determined by the extent of resource availability and degree of resource utilisation.[6]

Also, the paper reaffirms the well-established result that money-financed fiscal deficits undermine the sustainability of external debt (Dornbusch 1985; Worrell 2015: pp.24). The key channel relates to the idea that monetisation engenders a loss of foreign assets held by the central bank in a pegged regime (Frenkel and Johnson 1976), and nominal currency depreciation in a pure float

[5] Note that flow equilibrium is also omitted in analyses of entitlement obligations and public debt sustainability (Cerniglia et al. 2021).

[6] In Mian et al. (2021), fiscal space depends on aggregate demand, inequality and trend growth, which is partly consistent with the present study. However, their model does not admit resource availability, that is, net foreign assets held by the central bank, as an important driver of fiscal space.

(Branson 1977). These results explain why small open economies require support during the pandemic but also in the long term on consideration of pubic debt dynamics. They also explain why a sovereign issues debt in a foreign currency as monetisation undermines public debt sustainability. Though the channels differ, this finding is similar to Aguiar et al. (2014), where inflating away the debt increases the equilibrium interest rate and probability of default.

Related Literature. This paper contributes to several related literature. First, this work is closely related to a newly emerging scholarship on long-run fiscal deficits and public debt (Krugman 2020; Aspromourgos et al. 2010; Godley and Lavoie 2007; Kalecki 1943). The paper's contribution to this strand of the literature is that the underlying result holds in both exchange rate regimes and even when the economy is dynamically efficient. Krugman and Aspromourgos et al. propose the idea of a permanent fiscal deficit based on the prediction that dynamic inefficiency holds in the long run, and that the monetary authority influences the long-term yield on government bonds respectively. Kalecki—and Godley and Lavoie by way of simulation—show that a constant public debt and trade deficit ratio are consistent with a long-run fiscal deficit. Unlike these works, the present study accounts for both exchange rate regimes and the private sector's balance. Mian et al. (2021) and Reis (2021) are two recent studies that qualify the conditions of a permanent fiscal deficit or debt sustainability. Mian et al. formalise the idea that the rate of interest rises with the stock of debt, so primary deficits are only sustainable up to some threshold. In contrast, Reis argues that permanent fiscal deficits pay for themselves when a bubble premium exists, $m > r$, where m is the marginal product of capital. Crucially, primary deficits raise the bubble premium up to some threshold, after which the bubble pops due to a fall in r, and contracts the demand for sovereign debt. The key difference between these studies and the present work is that they do not admit stock-flow consistency. On the empirical front, the epigraph above summarises the recent evidence that long-run fiscal surpluses are the exception rather than the rule.[7]

Second, the standard approach to debt sustainability suggests that the government's initial domestic (external) debt stock must equal the net present value of its future primary fiscal (trade) surpluses (Burnside 2005a; Chuhan 2005; Blanchard and Weil 2001). I show that this approach also violates stock-flow consistency. More recently, Blanchard and Das (2017) posit that debt sus-

[7] It is worth highlighting the claim of expansionary fiscal austerity (Alesina et al. 2015; Giavazzi and Pagano 1990). It emphasises three channels: 1. An expectations channel of future tax reductions, 2. A financial channel that stimulates investment, and 3. An export-led channel through internal devaluation. See Botta (2020) for a formal proof that the claim is inconsistent with a lower debt ratio in the long run. Also, consult Yang et al. (2015) and Guajardo et al. (2014) for new empirical evidence of contractionary fiscal austerity.

tainability analyses omit the fact that the present value of net exports is a random variable, and that there is always an exchange rate depreciation that makes external debt sustainable (assuming an expansionary currency depreciation). The stock-flow model also derives similar results when a nominal currency depreciation is expansionary. However, newly emerging evidence suggests that an expansionary currency depreciation is the exception rather than the rule (Gopinath et al. 2020; Adler et al. 2020; Serana and Sousa 2017). In this case, the paper shows that a contractionary currency depreciation misses the bullseye if the fiscal rule is derived from the government's intertemporal budget constraint.

Third, Bocola and Lorenzoni (2020) and Worrell (2015) contend that a higher stock of foreign assets held by the central bank enhances fiscal capacity through currency market interventions and import cover respectively. In the former, a stable exchange rate stabilies the debt burden and in the latter, fiscal space increases when reserves exceed three months of import cover. However, the stock-flow model shows that reserves above this minimum do not unambiguously indicate a wider fiscal space, as the latter also depends on the degree of slack in the goods market. If this is ignored, fiscal policy becomes pro-cyclical and foreign exchange/debt crises are recurrent. Fourth, Ghosh et al. (2016) and Khemraj and Pasha (2012) highlight the empirical observation that inflation-targeting central banks also intervene in the foreign exchange market (so-called dual nominal anchors). The present work shows that only a stock-flow consistent fiscal rule stabilises the debt ratio and goods market at a constant exchange rate. This result suggests that excessive exchange rate volatility may be the price paid for stock-flow inconsistent fiscal rules.

Finally, the paper also relates to the well-established literature on optimal reserve balances (Obstfeld et al. 2010; Jeanne and Ranciere 2009; Worrell 1974). The present study indicates that the optimal stock of foreign reserves as a share of GDP rises with the primary fiscal deficit as a share of GDP, the weighted rate of interest, and external debt as a share of total government debt; but falls with economic growth and the private sector's stock of foreign savings and domestic debt as shares of GDP. The paper nests many of the established drivers of reserve accumulation—short-term external debt (the Greenspan-Guidotti rule), the output cost of a sudden stop (Jean and Ranciere), M2 (Obstfeld et al.), and exports, capital inflows, and government expenditures (Worrell).

The remainder of the paper is organised as follows. Section 2 presents some stylised facts on the cycles of foreign assets, exchange rates and fiscal deficits, and the theoretical model is introduced in Section 3. Section 4 concludes, and omitted proofs are presented in the Appendix.

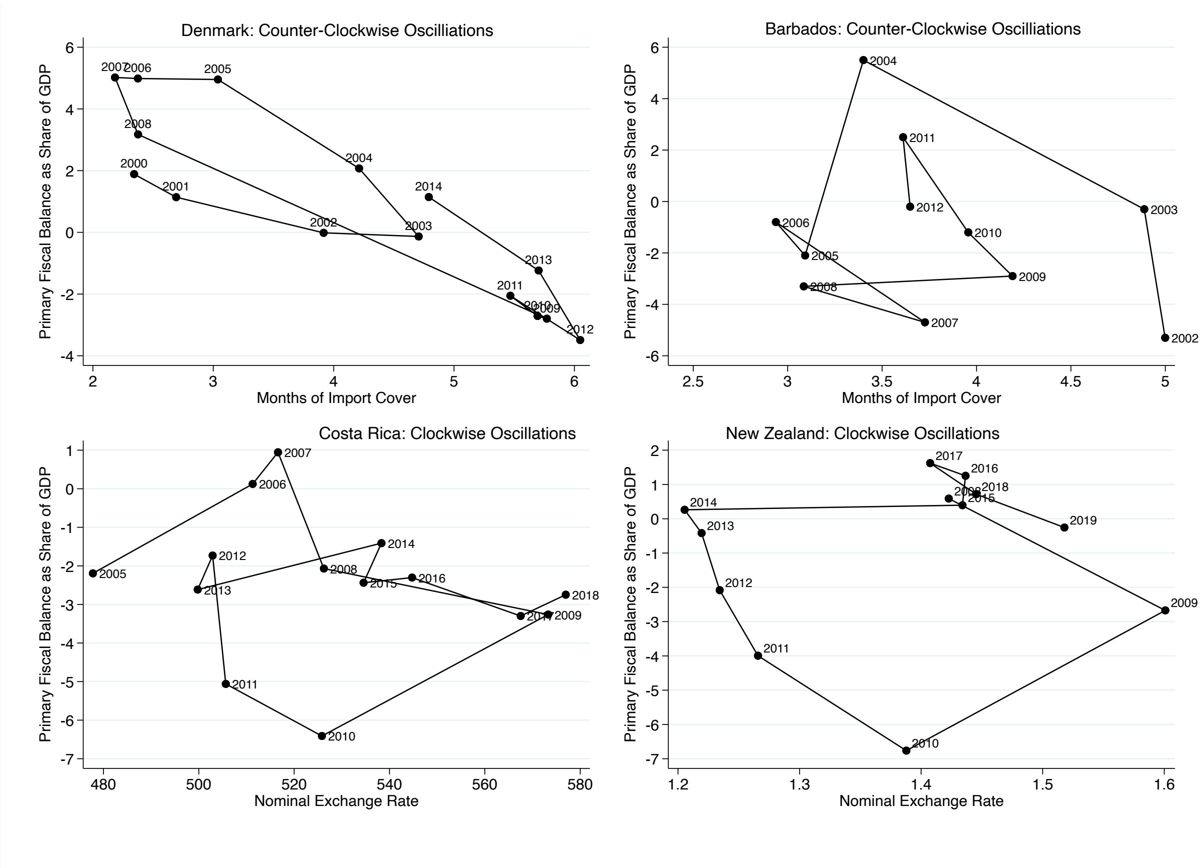
2 Stylised Facts

It is well established in both theory and evidence that open economies are subject to boom-bust cycles (Kohler 2019; Reinhart and Reinhart 2008), which usually involve significant currency depreciation, collapses in pegged exchange rate regimes, debt crises, and fiscal austerity (Garcia et al. 2018; Blanchard et al. 2013; Frenkel 2008; Mishkin 1999). Three important stylised facts emerge from the regularity of boom-bust cycles: 1. Fiscal policy tend to be pro-cyclical (Dzhambova 2021), 2. Exchange rates (real or nominal) are also pro-cyclical (Cordella and Gupta 2015), and 3. Central banks' foreign exchange reserve balances are counter-cyclical to maintain a fixed peg (Aizenman and Lee 2007). These stylised facts are reproduced in Figure 1, which shows the cyclical behaviour between the government's primary fiscal balance as a share of GDP and the nominal exchange rate/foreign assets held by the central bank in months of import cover. It illustrates the most recent one or two complete cycles that can be found in the data for four selected countries. The latter are randomly chosen for the purpose of illustration but the interested reader can derive similar results for other countries.

Two key results emerge. First, a fixed exchange rate regime—represented by Denmark and Barbados—has a counter-clockwise behaviour, which suggests that peaks/troughs in foreign assets held by the central bank or months of import cover precede peaks/troughs in the primary fiscal balance. This is intuitive as there is no exchange rate signal to indicate when fiscal policy deviates from fundamentals. Moreover, it suggests that when a fiscal crisis is realised, it is already too late as foreign assets held by the central bank have already peaked. It follows that the primary fiscal balance acts as a predator on foreign assets (the prey) in fixed exchange rate regimes. This implies that fiscal policy and central banks' foreign asset holdings are pro- and counter-cyclical respectively. Second, in a flexible exchange rate system—represented by Costa Rica and New Zealand—there are clockwise oscillations between the primary fiscal balance and the nominal exchange. Unlike the predator-prey dynamics in the fixed exchange rate system, peaks/troughs in the primary fiscal balance precede peaks/troughs in the nominal exchange rate. It follows that the latter acts as a constraint on fiscal policy, as pro-cyclical fiscal policy engenders pro-cyclical exchange rate fluctuations.

These stylised facts serve as the empirical motivation for the theoretical model that follows. What are the economic mechanisms that underpin the cyclical fluctuations? What do the oscillations imply for fiscal and debt sustainability on the one hand, and goods market equilibrium on the other? These are the motivating questions the model answers.

Figure 1: Primary Fiscal Balance and Nominal Exchange Rate/Import Cover



Notes: The data is sourced from the IMF, World Bank, and the OECD. Denmark and Barbados reflect the case of a fixed peg, while Costa Rica and New Zealand illustrate the case of a flexible exchange rate system. In the former case, the central bank's stock of foreign assets is transformed into months of import cover, where the minimum international recommendation is three months. Moreover, an increase in the nominal exchange rate indicates a depreciation relative to the USD. Further, a primary fiscal surplus and deficit are illustrated by a positive and negative primary fiscal balance respectively.

3 Model

This section introduces a stock-flow consistent, and continuous-time model of fiscal and debt sustainability in the cases of a fixed and flexible exchange rate regime.

3.1 The Case of a Fixed Exchange Rate Regime

This sub-section introduces the baseline model for the case of a fixed peg. It assumes that the nominal exchange rate ($\bar{q}$) is credibly fixed to the global reserve currency at a given rate, so $\bar{q}$ is omitted from the long-run analysis. The nominal exchange rate is expressed in local currency units per unit of the global reserve currency so that an increase in $\bar{q}$ indicates a nominal devaluation.

Consider Condition (1a), which states that the goods market clears at full capacity ($\bar{Y}$), a necessary requirement for long-run equilibrium. There is much debate about the dynamic adjustments that validate Condition (1a) but in the context of the long-run model it is sufficient to admit that it holds throughout.[8] This point is summarised in the following Assumption.

$$\bar{Y} = C + I + \tilde{G} + X - M \tag{1a}$$

Assumption 3.1 (Full Employment). *In the long-run steady state the economy obtains full employment such that the output gap or excess capacity (beyond the normal level) is zero.*

Given the previous Assumption, Condition (1a) can be written as follows:

$$\tilde{G} + X + I = T + M + S, \tag{1b}$$

where $\tilde{G}$, X and I are government outlays on goods/services and interest payments, exports and investment respectively, and T, M, and S are tax revenue, imports and savings respectively. Note that total government expenditure is defined as $\tilde{G} = G + rB$, where r is the real rate of interest, and B is the sum of domestic and foreign debt. Condition (1b) can be rewritten in terms of the government's primary fiscal balance as shown below:

[8] There are competing explanations or mechanisms through which Condition (1a) is validated. Old and new classical economics posit that excess capacity is eliminated through market forces (Prescott 1986), but Keynesians of all stripes contend that there is equilibrium excess capacity such that only fiscal stimulus and other interventions can restore full employment (Hein and Stockhammer 2010; Neary and Stiglitz 1983). Others claim that excess capacity engenders human capital flight (migration), inefficiency, fewer working hours, and a contraction in the labour force participation rate until excess capacity is eliminated (Cordeiro and Romero 2021; Porcile and Spinola 2018).

$$G-T=M-X+S-I-rB, \tag{2}$$

which demonstrates that external deficits $(M-X)$, and/or private sector savings $(S-I)$ engender primary deficits to attain goods market equilibrium; while higher interest payments require primary surpluses.[9] Note carefully that Condition (2) is equivalent to the government's *structural* primary balance or the *cyclically adjusted* primary balance given Assumption 3.1.

Condition (3) identifies the sources of funding for external deficits, where a dot $(\cdot)$ over a variable indicates its time derivative. Import receipts only exceed export earnings if the government $(\dot{B}_g^F)$, and/or the private sector $(\dot{B}_p^F)$ borrow foreign currency from the international market; and/or if the central bank $(\dot{F}_{cb})$, and/or the private sector $(\dot{F}_p)$ use their foreign assets to pay for imports. Note that the subscripts g, cb, and p refer to the government, central bank and private sector respectively, while the superscript F indicates that an asset/liability is denominated in foreign currency.

$$M-X=\dot{B}_g^F+\dot{B}_p^F-\dot{F}_{cb}-\dot{F}_p \tag{3}$$

A positive private sector savings balance is accumulated in the form of high-powered money issued by the central bank $(\dot{H})$, foreign currency assets $(\dot{F}_p)$, and/or government bonds issued in local currency units $(\dot{B}_g)$, less its accumulation of foreign $(\dot{B}_p^F)$ and domestic debt $(\dot{B}_p)$.

$$S-I=\dot{H}+\dot{F}_p+\dot{B}_g-\dot{B}_p^F-\dot{B}_p \tag{4}$$

Substitution of Equations (3) and (4) into (2) yields the government's primary fiscal balance where $\dot{B}_p^F$ and $\dot{F}_p$ cancel out:

$$G-T=(\dot{B}_g^F+\dot{B}_g)-\dot{F}_{cb}+\dot{H}-\dot{B}_p-rB.$$

This result shows that the government's primary fiscal balance is directly related to the public sector's accumulation of domestic and foreign debt, central bank's dissaving of foreign assets, and the acceleration of high-powered money, but inversely related to the private sector's accumulation of domestic debt and the government's interest payments. As noted earlier, goods market equilibrium requires a long-run primary fiscal deficit if the private sector accumulates domestic

[9]See Schlicht (2006) for a formal discussion of how the goods market equilibrium condition requires an inverse relationship between interest payments and government outlay on goods and services.

assets or saves. In such a case, the long-run primary deficit is funded by a combination of bonds, high-powered money and central bank reserves.

Following a stylised central bank balance sheet where assets include F_{cb} and government securities held by the central bank (B_g^{cb}); and liabilities compose of H and government deposits (D_g), the evolution of high powered money is given as follows. Condition (5) states that high powered money accelerates when the central bank accumulates foreign assets and government bonds but contracts when the government's deposit balance accelerates.

$$\dot{H} = \dot{F}_{cb} + \dot{B}_g^{cb} - \dot{D}_g \tag{5}$$

Given the fixed exchange rate regime, the following assumption specifies that the accumulation of foreign assets and deceleration in government deposit balance are perfectly sterilised by the central bank.

Assumption 3.2 (Complete Sterilisation). *The accumulation of foreign assets (F_{cb}) and reductions in government deposit balance (D_g) are completely sterilised or neutralised by the central bank's sale of government bonds (B_g^{cb}), so that $\dot{F}_{cb}$ and $\dot{D}_g$ have no effect on the dynamics of high-powered money ($\dot{H}$). Thus, consistent with the central bank's balance sheet constraint:* $\dot{F}_{cb} = \dot{D}_g - \dot{B}_g^{cb}$.

This assumption captures the basic premise of monetary policy in open economies with a credible peg, that is, intervention in the foreign exchange market and changes in government deposit balance should not affect the supply of high-powered money (Burnside et al. 2005; Frenkel and Johnson 1976). Otherwise, the market expects a currency devaluation when there is an excess supply of high-powered money, and the central bank loses control of its fixed exchange rate regime (Flood and Garber 1984; Krugman 1979).

Assumption 3.2 has two caveats that are worth noting. First, incomplete sterilisation is common practice in many small open economies.[10] This is realised when the central bank monetises fiscal expenditures, overestimates the demand for high-powered money, or holds too few government bonds relative to sterilisation requirements. In the context of this work, incomplete sterilisation is the *weak case* as I demonstrate below that its principal macroeconomic pathology is consistent with complete sterilisation—the *strong case*. Second, some central banks issue their own sterilisation bonds, say (B_{cb}), so that the evolution of high-powered money is given by $\dot{H} = \dot{F}_{cb} + \dot{B}_g^{cb} - \dot{D}_g -$

[10] See (IMF 2015: pp.18, 51-54) for a discussion on how poor cash flow management and liquidity forecasting ability are prevalent in low-income countries.

$\dot{B}_{cb}$. In the interest of simplicity and without any loss of generality this form of sterilisation is ruled out. When a central bank issues its own liabilities it usually holds few government bonds so that it is reasonable to assume that $\dot{B}_g^{cb} = 0$, which makes both forms of sterilisation equivalent. The use of B_{cb} is also consistent with the empirical regularity of incomplete sterilisation when there is an enforced debt ceiling that limits the stock of liabilities a central bank can issue, the demand for high-powered money is overestimated, or the central bank monetises fiscal expenditures by selling fewer sterilisation bonds.[11]

Given Assumption 3.2 it follows that $\dot{F}_{cb} = \dot{D}_g - \dot{B}_g^{cb}$, and substituting this result into Condition (5), it is transparent that the supply of high-powered money is constant as shown below. This result indicates that the market for high-powered money clears, and the stock of high-powered money is consistent with a credible fixed exchange rate regime.

$$\dot{H} = 0$$

Note that the above result and Assumption 3.2 imply that the uncovered interest rate parity condition adjusted for country risk holds in the long run. Otherwise, the economy observes offsetting capital flows, and since this is ruled out in the long period the following Assumption holds throughout.

Assumption 3.3 (Adjusted Uncovered Interest Rate Parity Condition). *In the long-run steady state, the adjusted uncovered interest rate parity condition holds.*

Since $\dot{H} = 0$, the government's primary fiscal balance becomes:

$$G - T = (\dot{B}_g^F + \dot{B}_g) - \dot{F}_{cb} - \dot{B}_p - rB. \tag{6}$$

The following Lemma specifies the well-established result that under a fixed exchange rate regime and complete sterilisation, monetised fiscal expenditures are operatlionalised through the stock of foreign assets held by the central bank.[12]

Lemma 3.1 (Money-financed Fiscal Deficits and Foreign Reserves). *In a fixed exchange rate regime with complete sterilisation, monetised fiscal expenditures, that is, central bank's accumulation of government bonds and/or deceleration in government deposits, have a direct and negative effect on the central bank's accumulation of foreign assets.*

[11] See Khemraj (2021) for further details.

[12] This is a standard result in the monetary approach to the balance of payments (Frenkel and Johnson 1976), where excess money balances reduce the central bank's stock of foreign assets (Flood and Garber 1984; Krugman 1979).

The basic intuition is as follows. To maintain the fixed exchange rate regime, a competent central bank is keen to ensure that its purchase of government bonds (or $\Downarrow \dot{D}_g$) does not engender a rapid supply of high-powered money. To that end, it adjusts its balance sheet by selling foreign currencies in the local market, thereby, sterilising any accumulation of high-powered money. As noted earlier, Assumption 3.2 is the strong case but the result is the same even if central bank under-sterilises (weak case). In the weak case, the private sector has excess holdings of high-powered money (given its demand) and adjusts its balance sheet accordingly, principally, by accumulating foreign assets ($\dot{F}_p$).[13] Consequently, the central bank is forced to sell its foreign reserves to anchor the market's expected exchange rate to the fixed peg. Note carefully that there is a non-trivial difference between the strong and weak cases. In the former, monetised fiscal expenditures lower the central bank's accumulation of foreign assets without any change in high-powered money, while in the weak case, monetised expenditures also accelerate the supply of high-powered money. It follows that the latter is a weak signal of monetised fiscal expenditures.

Recall that government's total public debt (B) is the sum of its domestic and foreign bond issues so that the following holds.

$$\dot{B} = \dot{B}_g^F + \dot{B}_g$$

Thus, Condition (6) can be rewritten in terms of total public debt:

$$G - T = \dot{B} - \dot{F}_{cb} - \dot{B}_p - rB, \tag{7}$$

and rearranging this result in terms of $\dot{B}$ derives the evolution of total government debt.

$$\dot{B} = (G - T) + \dot{F}_{cb} + \dot{B}_p + rB$$

The following Assumption specifies the no Ponzi condition that prevents the government from issuing debt to cover interest payments rB. Though Blanchard and Weil (2001) highlight the conditions under which a Ponzi scheme is feasible, this is plausibly ruled out in the case of the open economy, where foreign debt is denominated in a foreign currency. In other words, foreign creditors are less likely to provide loans to cover interest payments, and this constraint is especially binding if the domestic debt is continuously rolled over.

[13] See Cenedese and Elard (2021) for recent evidence of how quantitative easing (surplus liquidity) produces capital flight in advanced economies, and Marcel (2012) for evidence that surplus liquidity serves as an important push factor for capital in a wider class of economies.

Assumption 3.4 (No Ponzi Scheme Condition). *The government does not infinitely accumulate debt that is never repaid.*

Given Assumption 3.4, the evolution of total government debt becomes:

$$\dot{B} = (G - T) + \dot{F}_{cb} + \dot{B}_p. \tag{8}$$

This result shows that domestic and foreign bonds are *only* issued when $\dot{F}_{cb}$, $\dot{B}_p$, or the primary fiscal deficit increases.

Let $b = \frac{B}{P\bar{Y}}$ define the total government debt to GDP ratio, it follows that the stock of total government debt can be specified as shown below.

$$B = (b)P\bar{Y} \tag{9}$$

Taking the total differential of Definition (9) and dividing by nominal GDP ($P\bar{Y}$) yields the dynamics of total government debt to GDP ratio ($\dot{b}$):

$$\frac{\dot{B}}{P\bar{Y}} = \frac{\dot{b}P\bar{Y} + \dot{P}b\bar{Y} + \dot{\bar{Y}}bP}{P\bar{Y}},$$

which is simplified to the following, where $g = \dot{\bar{Y}}/\bar{Y}$ is the long-run rate of economic growth, and r is a weighted average of the real rate of interest on domestic and foreign debt after invoking the Fisher equation $\left(r = i - \frac{\dot{P}}{P}\right)$.[14] The weights are the respective shares of foreign ($\alpha_F = B_g^F/B$) and domestic debt ($1 - \alpha_F = B_g/B$) in total government debt.

$$\dot{b} = \frac{\dot{B}}{P\bar{Y}} + (r - g)b \tag{10}$$

Substitution of Condition (8) into (10) shows that the dynamics of total government debt are determined by the government's primary fiscal balance, dynamics of the central bank's foreign exchange reserve balances, accumulation of private sector debt; and the difference between the rate of interest and economic growth weighted by the total public debt as a share of GDP.

$$\dot{b} = \frac{G - T + \dot{F}_{cb} + \dot{B}_p}{P\bar{Y}} + (r - g)b \tag{11}$$

[14]When the Fisher equation is invoked, $\frac{\dot{B}}{P\bar{Y}}$ simplifies to the following:

$$\dot{b} = \frac{\dot{B}}{P\bar{Y}} + \left(i - \frac{\dot{P}}{P} - g\right)b.$$

This is the baseline model and its main results are presented in section 3.3.

3.2 The Domar Condition and Stock-Flow Inconsistency

This sub-section demonstrates that the Domar debt sustainability is not stock-flow consistent.

Following Equation (11)—the baseline model—the Domar condition is derived below when $\dot{F}_{cb} = \dot{B}_p = 0$, where $\Omega = \frac{G-T}{P\bar{Y}}$ is the government's primary fiscal balance as a share of GDP.

$$\dot{b} = \Omega + (r-g)b \tag{12}$$

The following outlines a formal definition of fiscal and public debt sustainability.

Definition 3.1 (Debt and Fiscal Sustainability). *When the government's total public debt as a share of GDP converges to a finite value (b^*) or a steady state such that $\dot{b} = 0$, public debt is sustainable. Any fiscal strategy that achieves $\dot{b} = 0$ is sustainable.*

The Domar condition for debt sustainability is summarised in the following Lemma.

Lemma 3.2 (Domar Debt Sustainability Condition). *When $r < g$, the debt ratio converges to a finite value, if and only if, the primary fiscal deficit as a share of GDP ($\Omega > 0$) is equal to $r < g$. Conversely, when $r > g$, government must incur a long-run primary fiscal surplus as a share of GDP ($\Omega < 0$) equal to $r > g$ to ensure debt sustainability.*

This result implies the following fiscal rule when $\dot{b} = 0$, where the subscript *DC* indicates that the rule is based on the Domar condition.

$$\Omega_{DC} = b(g-r) \tag{13}$$

This debt sustainability condition rests on special assumptions such that the fiscal rule Ω_{DC} may be consistent with long-run debt stock equilibrium, but not necessarily internal and external flow equilibria. It follows that the Domar condition violates the necessary stock-flow consistency of long-run equilibrium analysis. This point is formulated in the following proposition.

Proposition 3.1 (Domar Condition and Stock-Flow Inconsistency). *The Domar debt sustainability condition or its implied fiscal rule Ω_{DC} is not consistent with stock-flow equilibria.*

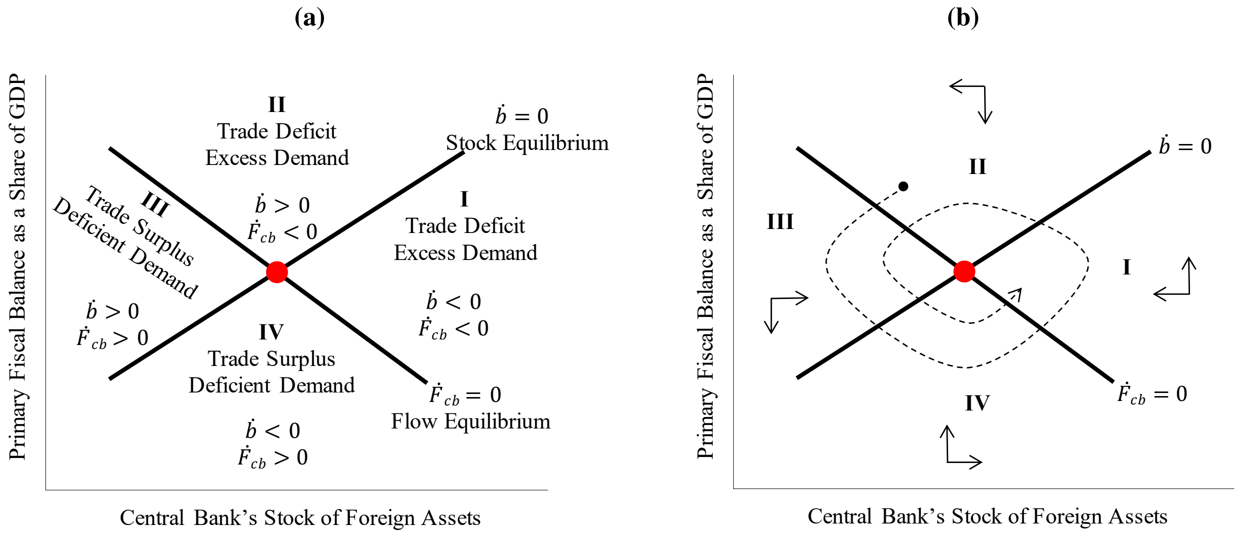
The basic intuition relates to the idea that stock-flow equilibria require a fiscal rule that is consistent with stable debt dynamics and full employment equilibrium. Since the Domar fiscal rule omits internal and external balance, its fiscal rule may generate excess or deficient aggregate demand—only by accident is its fiscal rule consistent with full employment equilibrium. It follows that there are necessarily upswings and downswings in the trajectory of total government debt as a share of GDP if the fiscal authority adheres to the Domar fiscal rule Ω_{DC}. The latter is akin to throwing darts and missing the bullseye, where the bullseye is the government's primary fiscal balance that attains stock-flow equilibria. This insight is summarised in the following Proposition.

Proposition 3.2 (A Debt Targeting Fiscal Rule and Cyclical Oscillations). *In a fixed exchange rate regime, the Domar fiscal rule Ω_{DC} necessarily overshoots and undershoots the primary fiscal balance and central bank's stock of foreign assets consistent with stock-flow equilibria.*

Figure 2 illustrates the basic idea. The upward sloping curve $\dot{b} = 0$ shows the combinations of the government's primary fiscal balance and the central bank's foreign reserves that are consistent with a stable or steady-state debt ratio (stock equilibrium). It is upward sloping because as the central bank's stock of foreign assets increases, so does long-run economic growth (g); which requires a bigger fiscal deficit to stabilise the public debt ratio. The basic intuition is as follows. Long-run growth is principally determined by innovation and technological change, which rely on imported capital and thereby, the central bank's stock of foreign assets. In turn, flow equilibrium is illustrated by the downward sloping locus $\dot{F}_{cb} = 0$, which depicts the various combinations of the government's primary fiscal balance and the central bank's foreign reserves that are consistent with goods market equilibrium at full employment. It is negatively sloped because a higher stock of foreign assets held by the central bank increases consumption and investment, which engender excess demand. Consequently, fiscal austerity is necessary to restore goods market equilibrium. Stock-flow equilibria are realised when $\dot{b} = \dot{F}_{cb} = 0$—the bullseye (red dot).

Figure 2a is divided into four quadrants and illustrates the various stock-flow disequilibria. For example, when the economy is located within quadrant I, the total public debt to GDP ratio is falling (stock disequilibrium), and there is rapid economic growth consistent with excess demand and a trade deficit (flow disequilibrium). Note carefully that these disequilibria require a *different fiscal response*—a bigger fiscal deficit is necessary to stabilise the public debt ratio—but fiscal austerity is required to stabilise the goods market. Since only one fiscal strategy can rule the day, cyclical fluctuations are inevitable in the debt and goods markets. It follows that any fiscal policy that adheres to the Domar fiscal rule Ω_{DC}, that is, a fiscal strategy of debt sustainability, produces counter-clockwise adjustments that overshoot and undershoot stock-flow equilibria as is shown in Figure 2b (consistent with the evidence in Figure 1). This insight can be put differently. The

Figure 2: Domar's Condition, Stock-Flow Inconsistency and Missing the Bullseye



Domar fiscal rule Ω_{DC} is *pro-cyclical*, which is easily verified by a close examination of Figure 2a. For example, Ω_{DC} requires a bigger fiscal deficit in quadrant I when the economy is overheating, but austerity in quadrant III when there is excess capacity.

It is instructive to recall Domar in his own words:

> From now on the heroic assumption is made that the stream of monetary expenditures will always be sufficient to maintain the national income at the maximum level established by the productive forces of the country (Domar 1944: pp.817).

Domar appropriately assumes that full employment holds in his long-run analysis, but the Domar debt condition is derived from the government's budget identity rather than a stock-flow consistent model. This insight leads to the following Corollary.

Corollary 3.1 (Government's Budget Identity and Stock-Flow Inconsistency). *Any debt sustainability condition derived from the government's budget identity is not stock-flow consistent.*

This is a striking result as it suggests that debt sustainability analyses derived from a government's intertemporal budget constraint (IBC) produce overshooting and undershooting of the stable debt trajectory as a matter of design. It is worth highlighting the subtle appeal of the IBC. Based on the Domar condition, a primary fiscal deficit stabilises debt only when the economy is dynamically inefficient, that is, $r < g$. Since this suggests that there is an over-accumulation of capital, which is ruled out as a long-run outcome, it follows that $r > g$ rules the day, and a long-run primary fiscal surplus is a necessary and sufficient requirement for debt sustainability. In the subsequent section, I provide formal proof that this result is invalid.

This discussion leads to the following Theorem.

Theorem 3.1 (Intertemporal Budget Constraint and Stock-Flow Inconsistency). *The fiscal rule implied by a government's intertemporal budget constraint is not consistent with stock-flow equilibria.*

As noted by Cochrane (2005), this result indicates that a government's budget constraint is useful for valuing or tracking the evolution of public debt but it is not a constraint or an instrument to formulate a meaningful fiscal rule.

3.3 Main Results I

This sub-section presents the main results of the baseline model (reproduced below for convenience). To provide a stock-flow consistent debt sustainability condition, dynamic specifications for the central bank's stock of foreign assets and private sector's stock of debt are formulated below.

$$\dot{b} = \frac{G - T + \dot{F}_{cb} + \dot{B}_p}{P\bar{Y}} + (r - g)b$$

Consider Equation (14a), which illustrates that the central bank accumulates foreign assets when its target stock of foreign reserves increases (F^T), where $1 < \gamma < 0$ captures the speed of adjustment. Conversely, as the central bank's stock of foreign reserves (F_{cb}) rises above its target level or demand, it unloads foreign assets onto the local market so that $\dot{F}_{cb} < 0$. This is necessary as the target stock of foreign assets must be consistent with the fixed exchange rate, so that maintaining the target reserve balance is key to anchor market expectations to the long-run peg. Ergo, the central bank's stock of foreign assets is dynamically stable $\left(\frac{d\dot{F}_{cb}}{dF_{cb}} < 0\right)$ with temporary adjustments of upswings and downswings. It is worth noting that while foreign exchange reserve crises do occur, stability is always imposed by bailouts and policy adjustments.

$$\dot{F}_{cb} = \gamma(F^T - F_{cb}) \tag{14a}$$

Equation (14b) specifies the determinants of the target stock of foreign assets, where g_{nx} is the growth rate of net exports, r_D and r_F are the domestic and foreign rate of interest respectively, $\alpha_F = B_g^F/B$ is the government's external debt as a share of total government debt, and $\bar{q}^e$ is the expected nominal devaluation. It follows that the central bank increases its demand for foreign reserves as the economy's trade surplus accelerates and the interest rate differential expands, where the latter captures capital account surpluses. Note that the domestic rate of interest is a mark-up on the foreign rate so that an interest rate differential reflects changes in the mark-up. Equation (14b) also demonstrates that the central bank increases its demand for foreign assets when the government's external debt stock increases ($B\alpha_F = B_g^F$)—this is necessary to service debt denominated in foreign currency. However, the central bank lowers its demand or target level of foreign assets when there is an expected devaluation ($\bar{q}^e > 0$). Note that when the market's expected exchange rate is anchored to the fixed peg $\bar{q}^e = 0$.

$$F^T = \rho_0 + \rho_1 g_{nx} + \rho_2(r_D - r_F) + \rho_3 B\alpha_F - \rho_4(\bar{q}^e) \tag{14b}$$

Equation (14c) presents a simple formulation of the market's expected exchange rate, which rises with the private sector's stock of foreign assets (F_p).

$$\bar{q}^e = \kappa_0 + \kappa_1 F_p \tag{14c}$$

In turn, the private sector accumulates domestic debt based on the following specification, where $1 < \delta < 0$, B_p^T is the private sector's target debt stock, and B_p is its actual stock of domestic debt. If the latter exceeds its target, the private sector issues fewer bonds and the reverse is also true. Thus, the private sector's debt stock is dynamically stable $\left(\frac{d\dot{B}_p}{dB_p} < 0\right)$. Indeed, private sector debt crises do emerge but these are inevitably dynamically stable processes when government assumes private sector liability through bail-outs and takeovers.

$$\dot{B}_p = \delta(B_p^T - B_p) \tag{15a}$$

The private sector increases its target debt stock when capacity utilisation $\left(u = Y/\bar{Y}\right)$ increases. The basic intuition is that as actual output rises relative to full capacity ($\bar{Y}$), firms endeavour to expand production capacity and increase their target debt stock or loan demand.

$$B_p^T = \omega_0 + \omega_1 u \tag{15b}$$

Substitution of Equations (14a-15b) into (11) yields an augmented-Domar condition (ADC) that is stock-flow consistent.

$$\dot{b} = \frac{G - T + \gamma\Big(\rho_0 + \rho_1 g_{nx} + \rho_2(r_D - r_F) + \rho_3 B\alpha_F - \rho_4\kappa_0 - \rho_4\kappa_1 F_p - F_{cb}\Big)}{P\bar{Y}} + \frac{\delta(\omega_0 + \omega_1 u - B_p)}{P\bar{Y}} + (r - g)b$$

The above result shows that in addition to the Domar condition (Equation 12), the dynamics of total public debt depend on the growth rate of net exports, the interest rate differential, government's external debt stock as a share of GDP, the private sector's and central bank's share of foreign assets, capacity utilisation, and the private sector's debt as a share of GDP. A close inspection of the above derivation shows that g_{nx}, $(r_D - r_F)$, and u have very small direct economic effects on the dynamics of total public debt as their impacts are divided by GDP. It follows that these factors are only economically significant through their effects on stock variables, namely, the central bank's stock of foreign assets and the private sector's stock of domestic debt. Therefore, for practical purposes, g_{nx}, $(r_D - r_F)$, and u can be omitted from the augmented-Domar condition as follows, where f_p and f_{cb} are foreign assets as shares of GDP held by the private sector and the central bank respectively, and b_p is the private sector's share of domestic debt.

$$\dot{b} = \Omega - \gamma\rho_4\kappa_1 f_p - \gamma f_{cb} - \delta b_p + \Big((r + \gamma\rho_3\alpha_F) - g\Big)b \tag{16}$$

Equation (16) indicates that a bigger primary fiscal deficit accelerates the total public debt ratio, while foreign assets held by the private sector and the central bank, and the private sector's domestic debt as shares of GDP induce decelerations in the share of total public debt. Moreover, the latter explodes when the weighted rate of interest plus the share of government's external debt in total public debt exceed the long-run rate of growth, and the reverse is true. The intuition is straightforward. As the central bank's stock of foreign assets as a share of GDP increases, it necessarily means there is a trade surplus (or long-term inflows like FDI), which provides for falling public debt dynamics as the government becomes a *net lender* in world markets (or enhances its ability to repay). The same result holds if the private sector's stock of foreign assets as a share of GDP rises—albeit a smaller effect ($\gamma\rho_4\kappa_1 < \gamma$)—as the private sector saves in foreign currency rather than local currency units. Similarly, as the private sector's domestic debt as a share of GDP expands so does domestic demand, and sectoral balance requires less accumulation of public sector domestic debt. In other words, as the private sector increases its own domestic liabilities it is not simultaneously accumulating public sector debt. Finally, the public debt ratio accelerates as the stock of external debt in total public debt increases. The driving force behind this result relates to the idea that more foreign assets are quarantined for servicing the external debt rather than accelerating long-run growth as α_F increases.

It is transparent that the baseline model is significantly different from the Domar condition as it accounts for both the private sector and external balances. The above result explains that there are two requirements that long-run economic growth must obtain for debt sustainability. Long-run growth is essential to generate revenue to service domestic debt but also important to earn foreign assets to satisfy external debt obligations. The latter point indicates that the consideration of stock-flow equilibria brings to bear the external growth requirements for debt sustainability. This is an important contrast to the Domar model as it fully addresses the currency mismatch risks that the Domar condition omits.

Recall that the Domar condition indicates that a long-run primary fiscal surplus is a necessary and sufficient requirement for debt sustainability when the economy is dynamically efficient, that is, $r > g$ (Lemma 3.2). The following Proposition invalidates this claim.

Proposition 3.3 (Dynamic Efficiency and the Augmented-Domar Condition). *When the economy is dynamically efficient such that $(r + \gamma\rho_3\alpha_F) > g$, debt sustainability is always realised with a primary fiscal deficit as a share of GDP, if and only if:*

(a) the sum of the foreign assets held by the private sector and the central bank, and the private sector's domestic debt as shares of GDP exceed the weighted interest rate-growth rate inequality: $-\gamma\rho_4\kappa_1 f_p - \gamma f_{cb} - \delta b_p > (r + \gamma\rho_3\alpha_F - g)b$.

This is a striking result and it demonstrates that a long-run primary fiscal surplus is only feasible if the weighted interest rate-growth rate inequality is implausibly large. There are several intuitive reasons why a primary fiscal surplus is not consistent with debt sustainability. First, a long-run primary surplus implies that the private sector accumulates debt as a long-run outcome, which is not consistent with private sector debt sustainability. The inevitable private sector debt crisis requires fiscal deficits for stabilisation purposes, which proves that both the public debt trajectory and the fiscal surplus were unsustainable. Second, a long-run primary surplus requires a long-run current account surplus, which is only possible if trading partners accept long-run current account deficits, and even the USA with its dollar privilege finds it increasingly difficult to accept this long-run outcome. History and current events are also a useful guide in this regard, and they suggest that external imbalances adjust through various forms of conflict, for example, war, protectionism, and financial crises.[15] It follows that for political economy reasons, primary surpluses do not provide for stable debt dynamics as it is a beggar-thy-neighbour policy with stringent political and economic limits—the case of Germany and the Eurozone crisis are recent examples (Storm and Naastepad 2015). Third, a long-run primary surplus as a beggar-thy-neighbour policy is also not feasible in very small open economies. In this case, there is a long-run accumulation of foreign assets by central banks, and given the developmental requirements—political economy factors compel smaller primary surpluses or outright deficits—lest the fiscal and monetary authorities endure charges of forcing a poor/developing economy to sit on a pot of gold. Fourth, while exogenous shocks like natural disasters and global economic crises do provide justification for long-run primary surpluses, these are also not sustainable in the long run. Assuming that the primary surpluses are consistent with foreign asset accumulation and private sector savings (a reasonable assumption), then the long-run outcome is a stagnation of domestic demand and an explosion of unemployment.[16] In other words, a long-run primary and external surplus with private sector savings necessarily contract the economy in the long run. It is not a stretch of the imagination to suggest that this is not a sustainable long-run outcome. Historical evidence, theory, and reason are not kind to the claim of a long-run primary surplus. Unlike the latter, primary deficits as a long-run outcome do not require any adjustments. Both history and theory suggest that the corresponding private sector accumulation of assets can continue indefinitely, unless interrupted by financial crises induced by speculation et cetera. But even these crises do not require primary surpluses. On consideration of stock-flow equilibria, there is absolutely nothing odd or *irresponsible* about a long-run primary deficit.

The following Proposition indicates that the augmented-Domar condition has a faster speed of convergence to its steady-state.

Proposition 3.4 (Half-Life and Speed of Convergence). *As compared to the Domar condition, the augmented-Domar condition (ADC) has a shorter half-life and thus, a shorter period of convergence to its long-run steady state.*

This result is driven by the share of external debt in total public debt α_F. It is intuitive that as the latter increases so does the currency mismatch risk, which provides for an upper limit on external debt and a faster speed of convergence to the long-run debt ratio.

[15] See Klein and Pettis (2020) for how global imbalances are underpinned by intra-country class conflict, and Delpeuch et al. (2021) for evidence that this leads to protectionism. The key point is that long-run primary surpluses are corrected to ameliorate intra-country class conflict.

[16] See Hein and Dodig (2015) for a discussion of this point using Germany as a recent example.

If there is nothing irresponsible about a long-run primary deficit, why do foreign exchange crises co-exist with primary deficits? The short answer is to blame the Domar fiscal rule Ω_{DC} but the following Theorem provides further details.

Theorem 3.2 (Fiscal Rules and Foreign Exchange Crises). *A fiscal rule that adheres to the Domar condition has a probability greater than zero of a foreign exchange crisis, while the augmented-Domar condition has a fiscal rule with a zero probability of foreign exchange crisis. Equation (17) presents the augmented-Domar fiscal rule (Ω_{ADC}) when $\dot{b} = 0$.*

$$\Omega_{ADC} = \gamma\rho_4\kappa_1 f_p + \gamma f_{cb} + \delta b_p + b(g - r - \gamma\rho_3\alpha_F) \tag{17}$$

Much is lost with the stock-flow inconsistency of the Domar condition, least of which is the credibility of fiscal authorities. This Theorem shows that recurrent or cyclical foreign exchange crises can be explained by a debt-targeting fiscal rule akin to the Domar condition (recall Figure 2). Suffice to say, any open economy that obeys the augmented-Domar fiscal rule Ω_{ADC} will avoid foreign exchange crises and realise more stable economic performance.

Remark 3.1. It is worth highlighting that the augmented-Domar fiscal rule avoids the difficulty of estimating the cyclically adjusted fiscal balance, since Ω_{ADC} is by definition the structural primary balance (recall Assumption 3.1 and Condition 2).

Equation (17) merits further discussion. It indicates that a larger primary fiscal deficit as a share of GDP is required to stabilise the public debt ratio when foreign assets and domestic liabilities as shares of GDP increase, and dynamic efficiency holds. Several factors produce this result. First, as the private sector increases its domestic liabilities as a share of GDP, it also increases its flow of savings to service this debt. Consequently, deficient demand emerges and a higher primary fiscal deficit as a share of GDP is required to stabilise both public debt stock and goods market equilibrium. Second, larger savings in the form of foreign assets as a share of GDP (private sector and central bank) also engender excess capacity, which justifies larger primary fiscal deficits to stabilise output and debt stock. It follows that the long-run *fiscal space* is jointly determined by *resource availability* and *resource utilisation*. This point leads to the following Definition.

Definition 3.2 (Fiscal Space). *Fiscal space refers to the extent of resource availability—central bank's foreign assets as a share of GDP—that provide for a primary fiscal balance consistent with a stable debt ratio, and the degree of resource utilisation—private sector's foreign assets and* *domestic liabilities as shares of GDP—that provide for a primary fiscal balance consistent with full employment equilibrium.*

This Definition clearly outlines why the Domar fiscal rule Ω_{DC} overshoots and undershoots the primary fiscal balance consistent with stock-flow equilibria. Since it only accommodates debt stock equilibrium, it violates the Definition of fiscal space and recommends a long-run primary fiscal balance *irrespective* of the degree of resource utilisation. Note carefully that in the open-economy context, fiscal space does not depend on the mobilisation of tax revenue denominated in local currency units. Further, an increase in the stock of foreign assets held by the central bank is *not* sufficient to expand the fiscal space. The latter also requires *need* or the under-utilisation of domestic resources (excess capacity). On this front, many open economies violate the Definition of fiscal space, thus, the cycle of foreign exchange crises.

The following Proposition specifies that the augmented-Domar fiscal rule Ω_{ADC} is the only proverbial game in town that maximises welfare.

Proposition 3.5 (Augumented-Domar Fiscal Rule and Welfare). *The augmented-Domar fiscal rule maximises welfare by simultaneously stabilising the public debt ratio and obtaining goods market equilibrium at potential output. All other fiscal rules are inferior.*

The augmented-Domar condition is also instructive because it presents a compelling rationale for why open economies should accumulate foreign assets in the long run. Further, unlike the arbitrary international recommendation that central banks should hold foreign assets to cover no less than three months of imports, the augmented-Domar condition provides a country-specific and concrete guide to policymakers. For example, if the fiscal authority plans to incur a fiscal deficit of seven percent of GDP over some time horizon, given the private sector's debt share, foreign savings, and the weighted interest rate-growth rate inequality forecasts, the central bank can calculate the precise stock of foreign assets as a share of GDP that it must hold for debt sustainability and goods market equilibrium at potential output. Needless to say, if the estimated stock of foreign assets is infeasible given existing stock and flow projections, the fiscal target should be revised. This result is summarised in the following Proposition.

Proposition 3.6 (Central Bank's Optimal Stock of Foreign Assets as a share of GDP). *The optimal stock of foreign assets a central bank should hold as a share of GDP is given as follows:*

$$f_{cb}* = \frac{\Omega - \rho_4 \kappa_1 f_p - \delta b_p + b(r + \rho_3 \alpha_F - g)}{\gamma}, \tag{18}$$

where $f_{cb}*$ is consistent with stable debt and goods market equilibrium at potential output.

This result shows that fiscal and monetary policy coordination is essential as fiscal deficits necessarily influence the optimal stock of foreign reserves, and indeed, the actual stock of central bank reserves (see Lemma 3.1). On the point of coordination, Sargent and Wallace (1981) contend that even tight monetary policy in the present necessarily becomes loose if the fiscal authority commits to current and future deficits given the demand constraint for government bonds. But in the context of the open economy, especially the small and very-open, the principal constraint is the stock of foreign reserves the central bank holds rather than a demand constraint for bonds *per se*. Unlike Sargent and Wallace (1981), following Propositions 3.3, 3.5 and Definition 3.2, a primary fiscal deficit as a share of GDP is a necessary long-run outcome but its size is constrained by the extent of resource utilisation and the central bank's foreign assets as a share of GDP. Fiscal authorities are well advised to coordinate their fiscal strategies with the monetary authority. Also, Proposition 3.6 indicates that the optimal stock of foreign reserves rises with the weighted rate of interest and external debt as a share of total government debt, but falls with economic growth and the private sector's external savings and domestic debt.

Recall Lemma 3.1 that money-financed fiscal deficits lower the stock of foreign assets held by the central bank, then it is apparent that money creation explodes the public debt to GDP ratio, and undermines debt sustainability. This result is summarised in the following Proposition.

Proposition 3.7 (Money Creation and Debt Sustainability). *Money-financed fiscal deficits undermine debt sustainability by lowering the central bank's stock of foreign assets as a share of GDP, thereby, compromising the government's ability to service external debt.*

This result is a rude reminder that there *no free lunch* in the open economy with a fixed peg. It also predicts chronic foreign exchange shortages and debt sustainability complications in the medium- to long-term if pandemic budgets are money-financed in open economies.

Remark 3.2. On consideration of brevity and space, Equation (16) omits two factors that should be accounted for in empirical models. First, remittances as a share of GDP provide for more stable debt dynamics as it increases the stock of foreign assets. Second, even a fixed peg is subject to trade-weighted exchange rate risks, which can accelerate the public debt ratio depending on the currency composition of external debt (Burnside 2005b). Needless to say, these factors also affect the augmented-Domar fiscal rule and the central bank's optimal stock of foreign assets.

3.4 The Case of a Flexible Exchange Rate Regime

This sub-section introduces a flexible exchange rate regime into the stock-flow consistent model.

Recall the following Condition, which now accounts for the nominal exchange rate q:

$$G - T = (q\dot{B}_g^F + \dot{B}_g) - q\dot{F}_{cb} + \dot{H} - \dot{B}_p - qrB,$$

and since F_{cb} is stationary in a pure float, the government's primary fiscal balance yields the following.

$$G - T = (q\dot{B}_g^F + \dot{B}_g) + \dot{H} - \dot{B}_p - qrB \tag{19}$$

Recall that government's total public debt is the sum of its domestic and foreign bond issues so that the following holds.

$$q\dot{B} = q\dot{B}_g^F + \dot{B}_g$$

Thus, Condition (19) can be rewritten in terms of total public debt:

$$G - T = q\dot{B} + \dot{H} - \dot{B}_p - qrB, \tag{20}$$

and rearranging this result in terms of $q\dot{B}$ derives the evolution of total government debt, which omitts qrB following Assumption 3.4 (the no-Ponzi scheme condition).

$$q\dot{B} = (G - T) + \dot{B}_p - \dot{H} \tag{21}$$

Let $b = \frac{qB}{P\bar{Y}}$ define the total government debt to GDP ratio, it follows that the stock of total government debt can be specified as shown below.

$$qB = (b)P\bar{Y} \tag{22}$$

Taking the total differential of Definition (22) and dividing by nominal GDP ($P\bar{Y}$) yields the dynamics of total government debt to GDP ratio ($\dot{b}$):

$$\frac{\dot{q}B}{P\bar{Y}} + \frac{q\dot{B}}{P\bar{Y}} = \frac{\dot{b}P\bar{Y} + \dot{P}b\bar{Y} + \dot{\bar{Y}}bP}{P\bar{Y}},$$

which is simplified to the following, where g_y is the long-run rate of economic growth, $g_q = \dot{q}/q$

is the long-run rate of nominal depreciation, and r is a weighted average of the real rate of interest on domestic and foreign debt after invoking the Fisher equation.

$$\dot{b} = \frac{q\dot{B}}{P\bar{Y}} + (r + g_q - g_y)b \tag{23}$$

Substitution of Condition (21) into (23) yields the dynamics of total government debt. In contrast to the case of a fixed exchange rate, the evolution of high-powered money and the long-run rate of nominal depreciation contracts and accelerates the public debt ratio respectively. Following portfolio balance models, money creation engenders a depreciation of the nominal exchange rate (Frankel 1983; Branson 1977), which indicates that it has competing effects on the dynamics of total government debt. In other words, if $\dot{H} > 0$, the long-run rate of nominal depreciation is accelerating, which undermines confidence in the currency as a store of value and medium of exchange. Since these are extreme events it is assumed that $\dot{H} = 0$ without any loss of generality.

$$\dot{b} = \frac{G - T + \dot{B}_p - \dot{H}}{P\bar{Y}} + (r + g_q - g_y)b \tag{24}$$

Substitution of Equations (15a) and (15b) into (24) derives the dynamic evolution of public debt—the analogous version of the fixed exchange rate regime—Equation (16). Note carefully that foreign assets held by the private sector and the central bank are omitted in this result, as these are now reflected in the long-run rate of nominal depreciation g_q. As the latter increases, the cost of purchasing foreign currencies to service external debt obligation increases in local currency units, consequently, the local currency burden of public debt accelerates. Moreover, since this is the case of a pure float, the central bank does not systematically accumulate foreign assets even if the share of external debt in total government debt is rising. The risk of currency mismatch is reflected in the long-run rate of nominal depreciation and the weighted rate of interest r.

$$\dot{b} = \Omega - \delta b_P + (r + g_q - g_y)b \tag{25}$$

Note that Equation (25) collapses into the stock-flow inconsistent Domar condition if $b_p = g_q = 0$. In this case, the Domar condition and its fiscal rule provide for an overshooting and undershooting of the stock-flow equilibria consistent with debt and exchange rate stability. The basic intuition relates to the idea that stock-flow equilibria require a fiscal rule that is consistent with stable debt dynamics on the one hand, and full employment equilibrium with a stable exchange rate on the other. The latter is necessary because goods market equilibrium (and debt stability) are not realised if the exchange rate is exploding or collapsing. This point is summarised in the following Proposition.

Proposition 3.8 (Debt-Targeting Fiscal Rule and Cyclical Oscillations: Pure Float). *When a nominal currency depreciation is contractionary, the Domar fiscal rule Ω_{DC} necessarily overshoots and undershoots the primary fiscal balance and nominal exchange rate consistent with stock-flow equilibria.*

Figure 3 illustrates the basic idea. The upward sloping curve $\dot{q} = 0$ shows the combinations of the government's primary fiscal balance and the nominal exchange rate that are consistent with goods market equilibrium at potential output. It is upward sloping because a nominal depreciation reduces demand, which requires a bigger primary fiscal deficit to stabilise the goods market (the case of contractionary depreciation). In the latter case, a nominal depreciation contracts the wage share and depresses consumption demand, and/or deteriorates the trade balance when domestic substitutes are unavailable or inadequate. In turn, stock equilibrium is illustrated by the downward sloping locus $\dot{b} = 0$, which depicts the various combinations of the government's primary fiscal balance and the nominal exchange rate that are consistent with a steady-state public debt ratio. It is negatively sloped because a nominal depreciation accelerates the public debt ratio, which requires fiscal austerity to restore debt sustainability. Stock-flow equilibria are realised when $\dot{b} = \dot{q} = 0$.

Figure 3: The Case of a Pure Float: Domar's Condition and Missing the Bullseye

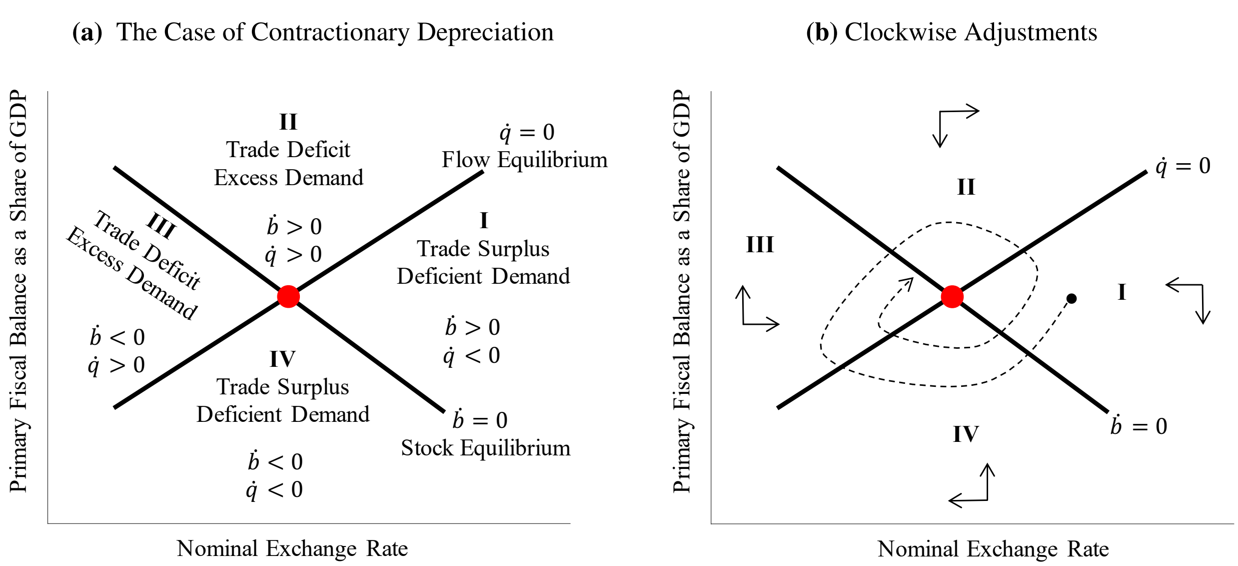


Figure 3a is divided into four quadrants and illustrates the various stock-flow disequilibria. For example, when the economy is located within quadrant I, the total public debt to GDP ratio is accelerating (stock disequilibrium), and there is a rapid appreciation of the nominal exchange rate, consistent with deficient demand and a trade surplus (flow disequilibrium). Crucially, these disequilibria require a *different fiscal response*—fiscal austerity is necessary to stabilise the public debt ratio—but a bigger fiscal deficit is required to stabilise the goods market and the nominal exchange rate. It follows that a debt-targeting fiscal rule necessarily produces cyclical fluctuations that overshoot and undershoot the nominal exchange rate and primary fiscal balance that are consistent with stock-flow equilibria. Figure 3b shows that these cyclical oscillations are clockwise adjustments (consistent with the evidence in Figure 1).

In sum, the Domar fiscal rule also misses the bullseye when a nominal currency depreciation is contractionary. However, a debt-targeting fiscal rule produces non-cyclical adjustments when a nominal currency depreciation is expansionary. This point is summarised in the following Proposition.

Proposition 3.9 (Debt-Targeting Fiscal Rule and Non-Cyclical Adjustments: Pure Float). *The Domar fiscal rule Ω_{DC} is akin to a stock-flow consistent fiscal rule when a nominal currency depreciation is expansionary.*

The basic intuition relates to the idea that an expansionary currency depreciation reduces the accumulation of public debt so that a bigger primary fiscal deficit is necessary for debt sustainability. Also, the expansionary currency depreciation produces an external surplus but contracts consumption as the currency depreciation lowers the wage share. Given the emerging trade surplus, the market expects a rapid appreciation of the nominal exchange rate, so that goods market equilibrium and exchange rate stability also require a bigger primary fiscal deficit. Note carefully that unlike Propositions 3.2 and 3.8, the fiscal response to stock-flow disequilibria is the same—a bigger primary fiscal deficit.

This is an important result but may only hold as a special case, as newly emerging evidence suggests that an expansionary currency depreciation is the exception rather than the rule.

3.5 Main Results II

This sub-section presents the main results of the baseline model in the case of a pure float (reproduced below for convenience).

$$\dot{b} = \Omega - \delta b_P + (r + g_q - g_y)b$$

The following Proposition outlines the condition for debt sustainability under the case of dynamic efficiency.

Proposition 3.10 (Dynamic Efficiency and Debt Sustainability: Pure Float). *When the economy is dynamically efficient such that $(r+g_q) > g_y$, debt sustainability is always realised with a primary fiscal deficit as a share of GDP, if and only if:*

(a) the private sector's domestic debt as a share of GDP exceeds the weighted interest rate-growth rate inequality: $-\delta b_p > (r+g_q-g_y)b$.

This is an interesting result and parallels Proposition 3.3 in that a long-run primary fiscal deficit is consistent with goods market equilibrium at potential output and sustainable public debt even in the case of a pure float. Of course, this does not suggest that a long-run primary fiscal surplus will never be observed. This is possible if the authorities decide to reduce the public debt ratio. However, recall the accompanying discussion to Proposition 3.3 that underscores the instabilities of a long-run primary fiscal surplus.

On consideration of a long-run depreciation of the nominal exchange rate, the following Proposition indicates that the augmented-Domar condition under a pure float has a faster speed of convergence to its steady state.

Proposition 3.11 (Half-Life and Speed of Convergence: Pure Float). *As compared to the Domar condition, the augmented-Domar condition has a shorter half-life and thus, a shorter period of convergence to its long-run steady state.*

This result indicates that a faster long-run depreciation of the nominal exchange rate either produces a debt/currency crisis in short order, or limits external borrowing by way of rapid convergence to the steady-state public debt ratio. Much hinges on the long-run dynamics of the exchange rate.

Equation (26) presents the augmented-Domar fiscal rule (Ω^{fxr}_{ADC}) when $\dot{b}=0$, where the superscript fxr indicates the flexible exchange rate regime.

$$\Omega^{fxr}_{ADC} = \delta b_p + b(g_y - r - g_q) \tag{26}$$

The fiscal rule Ω^{fxr}_{ADC} is stock-flow consistent as it accounts for the private sector and external balances. The former and the latter are reflected by the private sector's debt share (b_p) and the long-run depreciation of the nominal exchange rate respectively (g_q). Note that any excess demand, say, because of unwarranted accumulation of private sector liabilities, induces a depreciation of the nominal exchange rate that prevents an overshooting of the primary fiscal balance consistent with stock-flow equilibria. This point is summarised in the following Theorem.

Theorem 3.3 (Stock-Flow Consistent Fiscal Rule: Pure Float). *A fiscal rule that adheres to the augmented-Domar condition $\Omega_{ADC}^{f\,x\,r}$ provides for both debt and exchange rate stability.*

This is a striking result because it demonstrates that exchange rate volatility is the price paid for following a stock-flow inconsistent fiscal rule (recall Figure 3). In other words, in the case of a pure float, a debt-targeting fiscal rule produces enormous swings in the nominal exchange rate as a matter of design, which inevitably have feedback effects on the dynamics of public debt. Such volatility usually serves as a compelling rationale for fixed or managed exchange rate regimes, but Theorem 3.3 shows that the stock-flow consistent fiscal rule $\Omega_{ADC}^{f\,x\,r}$ is sufficient.

4 Conclusion

Debt sustainability analyses are fundamentally forward-looking exercises that involve forecasts and therefore, economic judgments about future interest and growth rates, nominal exchange rate, current account, and primary fiscal balances. Different combinations of these forecast variables are used in domestic and external debt sustainability analyses, where sustainability is defined as stationarity—a steady-state debt ratio relative to some threshold. Crucially, the forecast variables emerge from the government's intertemporal budget constraint, which ignores goods market (flow) equilibrium. It follows that the forecasts derive a fiscal rule that inevitably produces cyclical oscillations—overshooting and undershooting—around its own debt target or steady-state debt ratio. In other words, the standard debt sustainability analysis is not stock-flow consistent, which is necessary for appropriate analysis as debt ratios combine stock (debt) and flow (GDP or exports) variables. Though the cyclical fluctuations may eventually converge to a steady-state debt ratio, this comes at the price of significant volatility—debt and/or foreign exchange crises. Unsurprisingly, sovereign states frequently miss fiscal targets (Guzman and Heymann 2015), which are stock-flow inconsistent, and the fundamental reason why Blanchard et al. (2021) recommend abandoning fiscal rules for fiscal standards. The latter supposedly allows room for judgment.

This paper formalises a stock-flow consistent model of fiscal and debt sustainability, which shows that a primary fiscal deficit as a share of GDP obtains goods market equilibrium at potential output, and a steady-state debt ratio irrespective of the exchange rate regime, and even when the economy is dynamically efficient. This implies that a stock-flow consistent debt sustainability analysis undertakes forecasts to determine the *size* of the primary deficit consistent with a steady-state debt ratio, as opposed to the standard approach of forecasting the conditions that require a fiscal correction, that is, a primary surplus.

It is worthwhile to reconcile this result with historical evidence. In excellent historical research on public debt, Eichengreen et al. (2019) document the successful fiscal consolidation episodes in Great Britain, France, and the USA prior to 1913. Two points are in order. First, prior to 1913, debt instruments were not fundamental to the operation of monetary policy, where the central bank can influence the yield curve and provide for a long-run fiscal deficit. Second, deficit financing as a means to stabilise the goods market at full employment was not part of the economic orthodoxy until several years after the publication of The General Theory (Keynes 1936). This suggests that public debt through the ages was littered by periods of significant volatility and debt default. This history is carefully documented by Eichengreen et al. (2019). In related work, Mauro et al. (2015) employ a historical dataset that covers 55 countries for up to two hundred years and find evidence that the response of the primary fiscal surplus to variation in government debt is consistent with meeting governments' intertemporal budget constraint for the period prior to 2008. They document evidence of widespread primary fiscal surpluses in most advanced economies during the mid-1990s until at least the mid-2000s, and for emerging economies after the year 2000. Stock-flow equilibria require that these primary surpluses accelerate private sector indebtedness, which indeed has occurred and underpinned the recent global financial crisis (GFC) (Leigh et al. 2012; Mian and Sufi 2010). In my view, this best demonstrates that a long-run primary fiscal surplus is not sustainable as the economic consequences of the GFC require a primary deficit for stabilisation purposes.

Several policy implications are derived from this work. First, the finding that a long-run primary fiscal deficit as a share of GDP obtains *two* targets violates the Tinbergen Rule, which suggests that a policymaker requires n instruments to achieve n targets. Mason and Jayadev (2018) formally show in a closed economy model that the same unique combination of the fiscal balance and interest rate is consistent with output at potential and a constant public debt ratio, regardless of which instrument is assigned to which target. Crucially, irrespective of the instrument-target assignment, overshooting and undershooting of the stock-flow equilibria are inevitable. It follows that policymakers must aim for stock-flow consistency as the Tinbergen Rule produces economic cycles. Second, the paper derives fiscal rules that specify the precise primary fiscal deficit required for stock-flow equilibria, which are cognisant of the extent of resource availability (for example, foreign assets held by the central bank in the case of a pegged regime), and the degree of resource utilisation (extent of excess capacity). Third, the stock-flow consistent fiscal rule also provides for exchange rate stability as it stabilises debt and the nominal exchange rate consistent with flow equilibrium.

This work suggests several areas for future research. First, modeling the micro-foundations of the stock-flow equilibria is an area for further investigation. Second, future work should incorporate contingent liabilities and floating rates of interest on both domestic and external debt. Third, more work needs to be done to better understand the nexus among the public debt ratio, fiscal balance, and economic performance, albeit, in a stock-flow consistent and open-economy framework. Finally, this work is well placed to launch a new empirical literature on stock-flow consistent fiscal and debt sustainability analyses.

Appendix: Omitted Proofs

Proof of Lemma 3.1 From Assumption 3.2 the following holds:

$$\dot{F}_{cb} = \dot{D}_g - \dot{B}_g^{cb},$$

and it is straightforward that the central bank dishoards foreign assets when it increases its holdings of government bonds, and/or when the government's deposit balance decelerates. □

Proof of Lemma 3.2 This follows logically from Equation (12). □

Proof of Proposition 3.1 The Domar condition in Equation (12) is only realised when $\dot{B}_p = \dot{F}_{cb} = 0$, which implies that internal (private sector balance) and external (current account balance) flow equilibria are omitted. □

Proof of Proposition 3.2 This proof proceeds in two steps: 1. It derives the dynamic system that specifies the evolution of the government's primary fiscal balance and the stock of foreign assets held by the central bank, and 2. Evaluates the stability of the system and the properties of its dynamic adjustments.

Step 1. Consider the following dynamic adjustment of the primary fiscal balance, where Ω^T is the government's target primary balance and $0 < \mu < 1$ is an adjustment parameter.

$$\dot{\Omega} = \mu(\Omega^T - \Omega) \tag{A.1}$$

The government's target primary balance is determined by its debt targeting fiscal rule to obtain $\dot{b} = 0$.

$$\Omega^T \equiv \dot{b} = 0 \tag{A.2}$$

The Equation below reproduces the dynamic adjustment of the public debt ratio:

$$\dot{b} = \Omega + (r - g)b, \tag{A.3}$$

and Equation (A.4) shows that the long-run rate of economic growth is a positive function of the central bank's stock of foreign assets (F_{cb}). The basic intuition is as follows. Long-run growth is principally determined by innovation and technological change, which rely on imported capital and thereby, the central bank's stock of foreign assets.

$$g = \theta_0 + \theta_1 F_{cb} \tag{A.4}$$

Substitution of Equation (A.4) into (A.3) derives the primary fiscal balance that is consistent with a constant public debt ratio when $\dot{b} = 0$. Note that this result is the government's target primary balance.

$$\Omega^T \equiv \Omega = b(\theta_0 + \theta_1 F_{cb} - r) \tag{A.5}$$

Thus, $\dot{\Omega}$ is given as follows:

$$\dot{\Omega} = \mu\Big(b(\theta_0 + \theta_1 F_{cb} - r) - \Omega\Big). \tag{A.6}$$

In turn, Equation (A.7) shows the evolution of the central bank's stock of foreign assets, where g_{AD} is the growth rate of aggregate demand, and $0 < \nu < 1$ is an adjustment parameter. When the goods market clears $g = g_{AD}$, and deficient and excess demand are given by $g > g_{AD}$ and $g < g_{AD}$ respectively.

$$\dot{F}_{cb} = \nu(g - g_{AD}) \tag{A.7}$$

To derive the aggregate demand growth rate, consider the standard macroeconomic relationship:

$$1 = c + i + \Omega + nx, \tag{A.8}$$

where c, i, and nx are consumption, investment and net exports as shares of GDP respectively. The primary fiscal balance is taken as given and c, i, and nx are specified as follows. Note that these are simplistic specifications but sufficient for the task at hand. Each component of demand as a share of GDP is negatively related to the aggregate demand growth rate and positively related to the central bank's stock of foreign assets. The principal channel for the latter relates to the confidence that the pegged rate is well anchored. Note that growth in external demand z increases net exports as a share of GDP.

$$c = \psi_0 - \psi_1 g_{AD} + \psi_2 F_{cb} \tag{A.9}$$

$$i = \phi_0 - \phi_1 g_{AD} + \phi_2 F_{cb} \tag{A.10}$$

$$nx = \omega_0 + \omega_1 z - \omega_2 g_{AD} + \omega_3 F_{cb} \tag{A.11}$$

Substitution of Equations (A.9)-(A.11) into (A.8) derives the aggregate demand growth rate.

$$g_{AD} = \frac{\psi_0 + \phi_0 + \omega_0 - 1 + \omega_1 z + \Omega + F_{cb}(\psi_2 + \phi_2 + \omega_3)}{\psi_1 + \phi_1 + \omega_2} \tag{A.12}$$

In turn, substitution of Equations (A.4) and (A.12) into (A.7) yields the dynamic adjustment of the central bank's stock of foreign assets.

$$\dot{F}_{cb} = \nu\Big(\theta_0 + \theta_1 F_{cb} - \Big[\frac{\psi_0 + \phi_0 + \omega_0 - 1 + \omega_1 z + \Omega + F_{cb}(\psi_2 + \phi_2 + \omega_3)}{\psi_1 + \phi_1 + \omega_2}\Big]\Big) \tag{A.13}$$

Note that dynamic stability requires that $\theta_1 < \frac{-\psi_2 - \phi_2 - \omega_3}{\psi_1 + \phi_1 + \omega_2}$, otherwise, the central bank's stock of foreign assets explodes indefinitely, which is implausible. The basic intuition for this necessary condition is that as the central bank increases its reserve balance it improves confidence, which accelerates consumption and investment at a faster rate than the pace of foreign exchange-induced innovation.

The dynamic system is reproduced below.

$$\dot{\Omega} = \mu\Big(b(\theta_0 + \theta_1 F_{cb} - r) - \Omega\Big)$$

$$\dot{F}_{cb} = \nu\Big(\theta_0 + \theta_1 F_{cb} - \frac{\psi_0 + \phi_0 + \omega_0 - 1 + \omega_1 z + \Omega + F_{cb}(\psi_2 + \phi_2 + \omega_3)}{\psi_1 + \phi_1 + \omega_2}\Big)$$

Step 2. This system is evaluated by the terms of the Jacobian matrix.

$$J = \begin{bmatrix} \frac{d\dot{\Omega}}{d\Omega} & \frac{d\dot{\Omega}}{dF_{cb}} \\ \\ \frac{d\dot{F}_{cb}}{d\Omega} & \frac{d\dot{F}_{cb}}{dF_{cb}} \end{bmatrix} = \begin{bmatrix} -1 & b\theta_1 \\ \\ \frac{-1}{\psi_1+\phi_1+\omega_2} & \frac{-\psi_2-\phi_2-\omega_3}{\psi_1+\phi_1+\omega_2} + \theta_1 \end{bmatrix}$$

The trace of the Jacobian matrix is $-1 + \frac{-\psi_2-\phi_2-\omega_3}{\psi_1+\phi_1+\omega_2} + \theta_1 < 0$, and hence always negative—recall that $\theta_1 < \frac{\psi_2+\phi_2+\omega_3}{\psi_1+\phi_1+\omega_2}$. However, the determinant of the system is ambiguous as $\frac{\psi_2+\phi_2+\omega_3}{\psi_1+\phi_1+\omega_2} + \theta_1 - (b\theta_1)\Big(\frac{1}{\psi_1+\phi_1+\omega_2}\Big) \lesseqgtr 0$. The system is dynamically stable if the determinant is positive, otherwise, a saddle point (unstable) equilibrium exists. Since the product of the off-diagonal elements of the matrix is negative $(b\theta_1)\Big(\frac{-1}{\psi_1+\phi_1+\omega_2}\Big) < 0$, the system satisfies the necessary condition for oscillations or cyclical dynamics. Thus, a saddle-point can be ruled out, which implies that the system has a stable focus, consistent with the cyclical adjustments in Figure 2.

The positively sloped locus $\dot{b} = 0$ in Figure 2 is given by:

$$\Omega_{b*} = b(z + z_1 F_{cb} - r),$$

where an increase in F_{cb} accelerates long-run growth and contracts the public debt ratio, which requires a bigger fiscal deficit to stabilise debt. In turn, the negatively sloped curve $\dot{F}_{cb} = 0$ is shown below, where $\psi_2 + \phi_2 + \omega_3 > (\theta_1)(\psi_1 + \phi_1 + \omega_2)$.

$$\Omega_{fcb*} = (\psi_1 + \phi_1 + \omega_2)\theta_0 - \omega_1 z + 1 - \psi_0 - \phi_0 - \omega_0 - F_{cb}\Big(\psi_2 + \phi_2 + \omega_3 - (\theta_1)(\psi_1 + \phi_1 + \omega_2)\Big)$$

This result shows that as F_{cb} increases it generates excess demand in the goods market so that fiscal contraction is necessary for goods market equilibrium.

It is worth highlighting that though the system is dynamically stable, it is littered with periods of boom and bust, and a sufficiently deep bust is necessary to stabilise debt and foreign assets held by the central bank. This can be interpreted as the period when a sovereign approaches the IMF for assistance—this is the price paid for a stock-flow inconsistent fiscal rule. □

Proof of Corollary 3.1 Consider the government's budget identity expressed as a share of GDP, which ignores interest payments on debt.

$$\frac{G-T}{P\bar{Y}} = \frac{\dot{B}}{P\bar{Y}} + \frac{\dot{H}}{P\bar{Y}}$$

Since a credible exchange rate regime requires $\dot{H} = 0$, the identity reduces to the following.

$$\frac{G-T}{P\bar{Y}} = \frac{\dot{B}}{P\bar{Y}}$$

Recall that $\frac{\dot{B}}{P\bar{Y}}$ is given by

$$\frac{\dot{B}}{P\bar{Y}} = \frac{\dot{b}P\bar{Y} + \dot{P}b\bar{Y} + \dot{\bar{Y}}bP}{P\bar{Y}},$$

and after substitution into the government's budget identity, the stock-flow inconsistent Domar condition is derived below (following Proposition 3.1).

$$\dot{b} = \Omega + (r-g)b$$

□

Proof of Theorem 3.1 This follows logically from Proposition 3.1 and Corollary 3.1. □

Proof of Proposition 3.3 This follows logically from Equation (16). □

Proof of Proposition 3.4 This proof is straightforward and is shown below.

$$\overbrace{\frac{ln(2)}{\Big((r+\gamma\rho_3\alpha_F)-g\Big)}}^{\text{Augmented-Domar Condition}} < \overbrace{\frac{ln(2)}{(r-g)}}^{\text{Domar Condition}}$$

□

Proof of Theorem 3.2 Following Proposition 3.1, the Domar condition is not stock-flow consistent so that its implied fiscal rule Ω_{DC} engenders swings of deficient and excess demand, where the latter is consistent with primary deficits, losses of foreign assets and foreign exchange crises. From Lemma 3.2, is transparent that the Domar condition does not account for foreign assets and omits the risks of a currency mismatch. It follows that the Domar fiscal rule has a probability greater than zero that a foreign exchange crisis emerges. Moreover, the fiscal rule Ω_{ADC} is stock-flow consistent as it accounts for the private sector and external balances. It explicitly considers the risk of a currency mismatch by accounting for the stock of foreign assets as a share of GDP (private sector and central bank), and the share of external debt in total public debt. Further, Ω_{ADC} avoids excess and deficient demand in the goods market by accounting for the private sector's domestic liabilities as a share of GDP. It follows that Ω_{ADC} has a probability of zero that a foreign exchange crisis emerges. □

Proof of Proposition 3.5 This follows from Theorem 3.2, which implies that the augmented-Domar fiscal rule satisfies Definition 3.2. □

Proof of Proposition 3.6 This follows from Equation (16) when $\dot{b} = 0$. □

Proof of Proposition 3.7 Following Assumption 3.2 of complete sterilisation and Lemma 3.1, the central bank's stock of foreign assets as a share of GDP is given as follows.

$$f_{cb} = \frac{D_g}{P\bar{Y}} - \frac{B_g^{cb}}{P\bar{Y}}$$

It is immediately apparent that a decline in the government's deposit balance and/or an increase in the central bank's purchase of government bonds as shares of GDP lowers f_{cb}. From Equation (16), a decrease in f_{cb} accelerates the total public debt ratio:

$$\frac{\partial \dot{b}}{\partial f_{cb}} = -\gamma.$$

□

Proof of Proposition 3.8 This proof proceeds in two steps: 1. It derives the dynamic system that specifies the evolution of the government's primary fiscal balance and the nominal exchange rate when a currency depreciation is contractionary, and 2. Evaluates the stability of the system and the properties of its dynamic adjustments.

Step 1. Equations (A.14) and (A.15) show the cases of contractionary and expansionary depreciations respectively. There are several channels at work in the contractionary case. First, as a nominal currency depreciation depresses the economy, there are weaker learning-by-doing effects that undermine innovation and overall technological progress. Second, both the currency depreciation and the depressed economy lower the firm-level wage share to such an extent that there are disincentives to invest in labour-saving technology. Finally, a currency depreciation increases the costs of imported technology/capital in local currency units, and the cost of external finance (loan rate), both of which crowd out firm-level innovation expenditure. Consequently, the long-run rate of economic growth decreases (Ribeiro et al. 2016). These effects are reversed in the expansionary case so that a nominal currency depreciation increases long-run growth. In this case, the learning-by-doing effects and the lower firm-level wage share are sufficiently strong to induce technological progress and increase long-run growth.

$$g = \eta_0 - \eta_1 q, \text{ contractionary case} \tag{A.14}$$

$$g = \eta_0 + \eta_1 q, \text{ expansionary case} \tag{A.15}$$

Given Equations (A.1-A.3), the government's primary fiscal balance evolves as follows.

$$\dot{\Omega} = \mu\Big(b(\eta_0 - \eta_1 q - r) - \Omega\Big), \text{ contractionary case} \tag{A.16}$$

$$\dot{\Omega} = \mu\Big(b(\eta_0 + \eta_1 q - r) - \Omega\Big), \text{ expansionary case} \tag{A.17}$$

The nominal exchange rate evolves according to the following dynamic specification, where $0 < \lambda < 1$ is an adjustment parameter and q^e is the expected exchange rate. When the latter increases, this accelerates the rate of depreciation until the long-run rate adjusts to the market's expectation.

$$\dot{q} = \lambda(q^e - q) \tag{A.18}$$

When the goods market clears ($g_{AD} = g$) the market's expectation is anchored to the long-run trend. However, deficient ($g_{AD} < g$) and excess demand ($g_{AD} > g$) appreciates and depreciates the nominal exchange rate respectively. This is summarised below.

$$q^e = \kappa_0 + \kappa_1(g_{AD} - g) \tag{A.19}$$

Next, I formulate the growth rate of aggregate demand in the contractionary and expansionary cases. Recall that c, i, and nx are specified as follows with the inclusion of the nominal exchange. It is transparent that the latter has contractionary effects on c and nx but increases i. Note that when a nominal currency depreciation is contractionary $\psi_2 + \omega_3 > \phi_2$.

$$c = \psi_0 - \psi_1 g_{AD} - \psi_2 q \tag{A.20a}$$

$$i = \phi_0 - \phi_1 g_{AD} + \phi_2 q \tag{A.20b}$$

$$nx = \omega_0 + \omega_1 z - \omega_2 g_{AD} - \omega_3 q \tag{A.20c}$$

The following outlines the case of an expansionary depreciation, where $\omega_3 + \phi_2 > \psi_2$. In this case, a currency depreciation increases nx and investment demand relative to consumption. See Ribeiro et al. (2017) for a discussion on the various channels through which a currency depreciation produces contractionary/expansionary effects on aggregate demand.

$$c = \psi_0 - \psi_1 g_{AD} - \psi_2 q \tag{A.21a}$$

$$i = \phi_0 - \phi_1 g_{AD} + \phi_2 q \tag{A.21b}$$

$$nx = \omega_0 + \omega_1 z - \omega_2 g_{AD} + \omega_3 q \tag{A.21c}$$

Substitution of Equations (A.20a-A.20c) into (A.8) derives the aggregate demand growth rate in the contractionary case.

$$g_{AD} = \frac{\psi_0 + \phi_0 + \omega_0 - 1 + \omega_1 z + \Omega - q(\psi_2 + \omega_3 - \phi_2)}{\psi_1 + \phi_1 + \omega_2}, \text{ contractionary case} \tag{A.22}$$

In turn, substitution of Equations (A.21a-A.21c) into (A.8) derives the aggregate demand growth rate in the expansionary case.

$$g_{AD} = \frac{\psi_0 + \phi_0 + \omega_0 - 1 + \omega_1 z + \Omega + q(\phi_2 + \omega_3 - \psi_2)}{\psi_1 + \phi_1 + \omega_2}, \text{ expansionary case} \tag{A.23}$$

Next, I formulate the dynamic adjustment of the nominal exchange rate. Substitution of Equations (A.14) and (A.22) into (A.19) and (A.18) yields the following.

$$\dot{q} = \lambda \left(\kappa_0 + \kappa_1 \left[\frac{\psi_0 + \phi_0 + \omega_0 - 1 + \omega_1 z + \Omega - q(\psi_2 + \omega_3 - \phi_2)}{\psi_1 + \phi_1 + \omega_2} - \eta_0 + \eta_1 q \right] - q \right), \text{ contractionary case} \tag{A.24}$$

The dynamic system for the contractionary case (Figure 3) is reproduced below.

$$\dot{\Omega} = \mu \left(b(\eta_0 - \eta_1 q - r) - \Omega \right), \text{ contractionary case}$$

$$\dot{q} = \lambda \left(\kappa_0 + \kappa_1 \left[\frac{\psi_0 + \phi_0 + \omega_0 - 1 + \omega_1 z + \Omega - q(\psi_2 + \omega_3 - \phi_2)}{\psi_1 + \phi_1 + \omega_2} - \eta_0 + \eta_1 q \right] - q \right), \text{ contractionary case}$$

Step 2. This system is evaluated by the terms of the Jacobian matrix.

$$J = \begin{bmatrix} \frac{d\dot{\Omega}}{d\Omega} & \frac{d\dot{\Omega}}{dq} \\ \\ \frac{d\dot{q}}{d\Omega} & \frac{d\dot{q}}{dq} \end{bmatrix} = \begin{bmatrix} -1 & -\eta_1 \\ \\ \frac{1}{\psi_1+\phi_1+\omega_2} & \frac{-\psi_2-\omega_3+\phi_2}{\psi_1+\phi_1+\omega_2} + \eta_1 - 1 \end{bmatrix}$$

The trace of the Jacobian matrix is always negative as $-1 + \frac{-\psi_2-\omega_3+\phi_2}{\psi_1+\phi_1+\omega_2} + \eta_1 - 1 < 0$. However, the determinant of the system is ambiguous as $\frac{\psi_2+\omega_3+\phi_2}{\psi_1+\phi_1+\omega_2} + \eta_1 - 1 - (\eta_1)\left(\frac{1}{\psi_1+\phi_1+\omega_2}\right) \lesseqgtr 0$. Since the product of the off-diagonal elements of the matrix is negative $(-\eta_1)\left(\frac{1}{\psi_1+\phi_1+\omega_2}\right) < 0$, the system satisfies the necessary condition for oscillations or cyclical dynamics. Thus, a saddle-point can be ruled out, which implies that the system has a stable focus, consistent with the cyclical adjustments in Figure 3.

The positively sloped locus $\dot{q} = 0$ in Figure 3 is given by:

$$\Omega_q* = (\psi_1 + \phi_1 + \omega_2)(q - \eta_1 q + \eta_0 - \kappa_0) + q(\psi_2 + \omega_3 - \phi_2) - \psi_0 - \phi_0 - \omega_0 + 1 - \omega_1 z,$$

where an increase in q lowers the growth of aggregate demand and requires a bigger fiscal deficit to stabilise the goods market. Recall that $\psi_2 + \omega_3 > \phi_2$ and it is transparent that $q > \eta_1 q$. In turn, the negatively sloped locus $\dot{b} = 0$ is shown below, where an increase in q accelerates the public debt ratio and requires a fiscal contraction for debt sustainability.

$$\Omega_{b*} = b(\eta_0 - \eta_1 q - r)$$

Notwithstanding the stability of the dynamic system, a stock-flow inconsistent or debt-targeting fiscal rule induces significant volatility in the nominal exchange. It is also worth noting that the cyclical oscillations may impose stability by way of currency and debt crises that necessarily require fiscal adjustments. □

Proof of Proposition 3.9 This proof proceeds in two steps: 1. It derives the dynamic system that specifies the evolution of the government's primary fiscal balance and the nominal exchange rate when a currency depreciation is expansionary, and 2. Evaluates the stability of the system and the properties of its dynamic adjustments.

Step 1. Substitution of Equations (A.15) and (A.23) into (A.19) and (A.18) yields the dynamic adjustment of the nominal exchange rate.

$$\dot{q} = \lambda\left(\kappa_0 + \kappa_1\left[\frac{\psi_0 + \phi_0 + \omega_0 - 1 + \omega_1 z + \Omega + q(\phi_2 + \omega_3 - \psi_2)}{\psi_1 + \phi_1 + \omega_2} - \eta_0 - \eta_1 q\right] - q\right), \text{ expansionary case} \tag{A.25}$$

The dynamic system for the expansionary case is reproduced below.

$$\dot{\Omega} = \mu\left(b(\eta_0 + \eta_1 q - r) - \Omega\right), \text{ expansionary case}$$

$$\dot{q} = \lambda\left(\kappa_0 + \kappa_1\left[\frac{\psi_0 + \phi_0 + \omega_0 - 1 + \omega_1 z + \Omega + q(\phi_2 + \omega_3 - \psi_2)}{\psi_1 + \phi_1 + \omega_2} - \eta_0 - \eta_1 q\right] - q\right), \text{ expansionary case}$$

Step 2. This system is evaluated by the terms of the Jacobian matrix.

$$J = \begin{bmatrix} \frac{d\dot{\Omega}}{d\Omega} & \frac{d\dot{\Omega}}{dq} \\ \\ \frac{d\dot{q}}{d\Omega} & \frac{d\dot{q}}{dq} \end{bmatrix} = \begin{bmatrix} -1 & \eta_1 \\ \\ \frac{1}{\psi_1+\phi_1+\omega_2} & \frac{\phi_2+\omega_3-\psi_2}{\psi_1+\phi_1+\omega_2} - \eta_1 - 1 \end{bmatrix}$$

The trace of the Jacobian matrix is always negative as $-1 + \frac{-\psi_2-\omega_3+\phi_2}{\psi_1+\phi_1+\omega_2} + \eta_1 - 1 < 0$. However, the determinant of the system is ambiguous as $\frac{\phi_2+\omega_3-\psi_2}{\psi_1+\phi_1+\omega_2} - \eta_1 - 1 - (\eta_1)\left(\frac{1}{\psi_1+\phi_1+\omega_2}\right) \lesseqgtr 0$. Since the product of the off-diagonal elements of the matrix is unambiguously positive $(\eta_1)\left(\frac{1}{\psi_1+\phi_1+\omega_2}\right) > 0$, the system does not satisfy the necessary condition for oscillations or cyclical dynamics. Thus, one of two equilibria is possible: 1. A saddle-point (unstable) equilibrium if the determinant is negative or, 2. A stable node if the determinant is positive. Figure 4 illustrates both possibilities, where a stable equilibrium (stable node) is only possible if the $\dot{b} = 0$ locus is flatter than the $\dot{q} = 0$ locus, otherwise, a saddle-point equilibrium is realised.

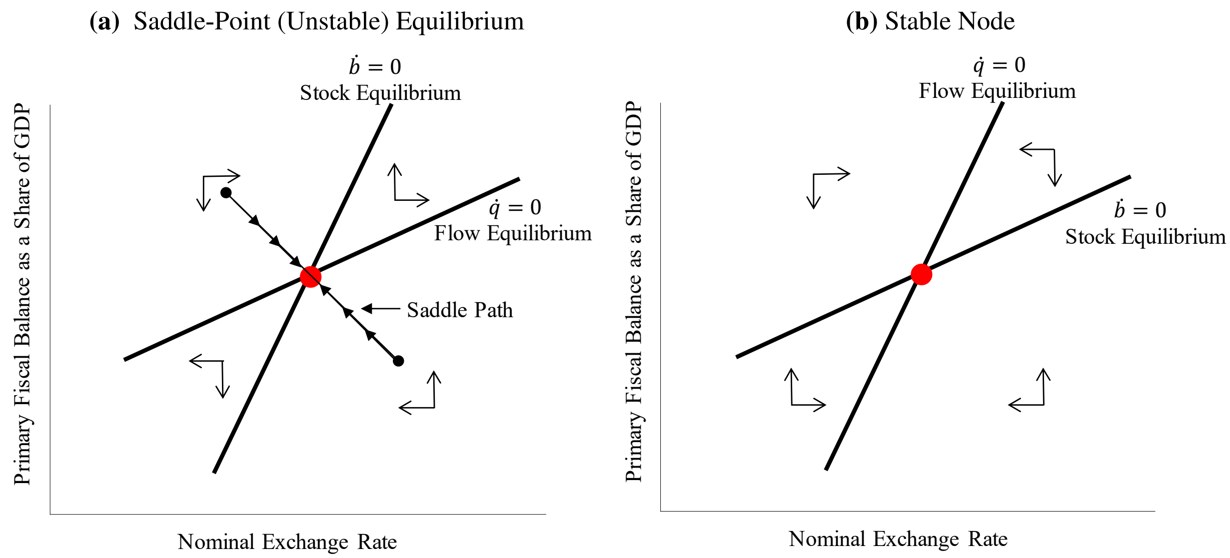
The positively sloped locus $\dot{q} = 0$ in Figure 4 is given by:

$$\Omega_q* = (\psi_1 + \phi_1 + \omega_2)(q + \eta_1 q + \eta_0 - \kappa_0) - q(\phi_2 + \omega_3 - \psi_2) - \psi_0 - \phi_0 - \omega_0 + 1 - \omega_1 z,$$

where $q + \eta_1 q > -q(\phi_2 + \omega_3 - \psi_2)$. It follows that an expansionary currency depreciation requires a bigger fiscal deficit to stabilise the goods market and the nominal exchange rate on account of a trade surplus and the market's expectation of a currency appreciation. In turn, the positively sloped locus $\dot{b} = 0$ is shown below, where an expansionary depreciation accelerates growth and reduces the public debt ratio, which requires a bigger fiscal deficit for debt sustainability.

$$\Omega_{b*} = b(\eta_0 + \eta_1 q - r)$$

Figure 4: The Case of an Expansionary Currency Depreciation



□

Proof of Proposition 3.10 This follows logically from Equation (25). □

Proof of Proposition 3.11 This proof is straightforward and is shown below.

$$\overbrace{\frac{ln(2)}{\left((r+g_q)-g_y\right)}}^{\text{Pure Float: Augmented-Domar Condition}} < \overbrace{\frac{ln(2)}{(r-g)}}^{\text{Domar Condition}}$$

□

Proof of Theorem 3.3 This follows logically from Equation (26). Any primary fiscal balance that deviates from Ω_{ADC}^{fxr} is consistent with stock-flow disequilibria, such that public debt and the nominal exchange rate are either exploding or decelerating. □

References

Adler, G., Casas, C., Cubeddu, L., Gopinath, G., Li, N., Meleshchuk, S., Buitron, C., Puy, D., and Timmer, Y. (2020). Dominant Currencies and External Adjustment. International Monetary Fund (IMF) Staff Discussion Note, SDN/20/05.

Aguiar, M., Amador, M., Farhi, E., and Gopinath, G. (2014). Sovereign Debt Booms in Monetary Unions. *American Economic Review: Papers and Proceedings*, 104(5):101–106.

Aizenman, J. and Lee, J. (2007). International Reserves: Precautionary versus Mercantilist Views, Theory and Evidence. *Open Economies Review*, 18(2):191–214.

Alesina, A., Favero, C., and Giavazzi, F. (2015). The Output Effects of Fiscal Consolidation Plans. *Journal of International Economics*, 96(1):S19–S42.

Aspromourgos, T., Rees, D., and White, G. (2010). Public Debt Sustainability and Alternative Theories of Interest. *Cambridge Journal of Economics*, 34(3):433–447.

Baldwin, R. and Weder di Mauro, B. (2020). *Mitigating the COVID Economic Crisis: Act Fast and Do Whatever It Takes*. CEPR Press.

Ball, L., Douglas, E., and Mankiw, N. G. (1998). The Deficit Gamble. *Journal of Money, Credit and Banking*, 30(4):699–720.

Barret, P., Das, S., Magisgretti, G., Pugacheva, E., and Wingender, P. (2021). After-Effects of the COVID-19 Pandemic: Prospects for Medium-Term Economic Damage. International Monetary Fund (IMF), Working Paper WP/21/203.

Blanchard, O. (2019). Public Debt and Low Interest Rates. *American Economic Review*, 109(4):1197–1229.

Blanchard, O. and Das, M. (2017). A New Index of Debt Sustainability. National Bureau of Economic Research, NBER Working Paper No. 24068.

Blanchard, O., Griffiths, M., and Gruss, B. (2013). Boom, Bust, Reovery: Forensics of the Latvia Crisis. *Brookings Papers on Economic Activity*, 2:325–371.

Blanchard, O., Leandro, A., and Zettelmeyer, J. (2021). Redesigning EU Fiscal Rules: From Rules to Standards. *Economic Policy*, Forthcoming.

Blanchard, O. and Weil, P. (2001). Dynamic Efficiency, the Riskless Rate, and Debt Ponzi Games under Uncertainty. *The B.E. Journal of Macroeconomics*, 1(2):1–23.

Bocola, L. and Lorenzoni, G. (2020). Financial Crises, Dollarization, and Lending of Last Resort in Open Economies. *American Economic Review*, 110(8):2524–2557.

Botta, A. (2020). The Short- and Long-Run Inconsistency of the Expansionary Austerity Theory: A Post-Keynesian/Evolutionist Critique. *Journal of Evolutionary Economics*, 30(1):143–177.

Branson, W. H. (1977). Asset Markets and Relative Prices in Exchange Rate Determination. *Sozialwissenschafiliche Annalen*, 1(69-89).

Burnside, C. (2005a). An Introduction to Fiscal Sustainability in Theory and Practice. In Burnside, C., editor, *Fiscal Sustainability in Theory and Practice: A Handbook*. The International Bank for Reconstruction and Development / The World Bank.

Burnside, C. (2005b). Some Tools for Fiscal Sustainability Analysis. In Burnside, C., editor, *Fiscal Sustainability in Theory and Practice: A Handbook*. The International Bank for Reconstruction and Development / The World Bank.

Burnside, C., Eichenbaum, M., and Rebelo, S. (2005). Currency Crises and Fiscal Sustainability. In Burnside, C., editor, *Fiscal Sustainability in Theory and Practice: A Handbook*. The International Bank for Reconstruction and Development / The World Bank.

Canofari, P., Piergallini, A., and Piersanti, G. (2020). The Fallacy of Fiscal Discipline. *Macroeconomic Dynamics*, 24(1):55–68.

Cenedese, G. and Elard, I. (2021). Unconventional Monetary Policy and the Portfolio Choice of International Mutual Funds. *Journal of International Money and Finance*, 115.

Cerniglia, F., Dia, E., and Hallett, A. (2021). Fiscal Sustainability under Entitlement Spending. *Oxford Economic Papers*, 73(3):1175–1199.

Chuhan, P. (2005). Debt and Debt Indicators in the Measurement of Vulnerability. In Burnside, C., editor, *Fiscal Sustainability in Theory and Practice: A Handbook*. The International Bank for Reconstruction and Development / The World Bank.

Cochrane, J. (2005). Money as a Stock. *Journal of Monetary Economics*, 52(3):501–528.

Cordeiro, A. and Romero, J. (2021). Reconciling Supply and Demand: New Evidence on the Adjustment Mechanisms between Actual and Potential Growth Rates. *Review of Political Economy*.

Cordella, T. and Gupta, P. (2015). What Makes a Currency Procyclical? An Empirical Investigation. *Journal of International Money and Finance*, 55:240–259.

Delpeuch, S., Fize, E., and Martin, P. (2021). Trade Imbalances and the Rise of Protectionism. Centre for Economic Policy Research, CEPR Discussion Paper 15742.

Domar, E. (1944). The Burden of the Debt and the National Income. *The American Economic Review*, 34(4):798–827.

Dornbusch, R. (1985). External Debt, Budget Deficits, and Disequilibrium Exchange Rates. In Smith, G. and Cuddington, J., editors, *International Debt and the Developing Countries*. Washington, D.C: World Bank.

Dzhambova, K. (2021). "When it Rains, it Pours": Fiscal Policy, Credit Constraints and Business Cycles in Emerging and Developed Economies. *Journal of Macroeconomics*, 69.

Eichengreen, B., El-Ganainy, A., Esteves, R., and Mitchener, K. (2019). Public Debt through the Ages. In Abbas, S., Pienkowski, A., and Rogoff, K., editors, *Sovereign Debt: A Guide for Economists and Practitioners*. Oxford University Press.

Escolano, J., Shabunina, A., and Woo, J. (2017). The Puzzle of Persistently Negative Interest Rate-Growth Differentials: Financial Repression or Income Catch-Up? *Fiscal Studies*, 38(2):179–217.

Flood, R. and Garber, P. (1984). Collapsing Exchange Rate Regimes: Some Linear Examples. *Journal of International Economics*, 17(1-2):1–13.

Frankel, J. A. (1983). Monetary and Portfolio-Balance Models of Exchange Rate Determination. In Bhandari, J., editor, *Economic Interdependence and Flexible Exchange Rates*. Cambridge, MA: MIT Press.

Frenkel, J. A. and Johnson, H. G., editors (1976). *The Monetary Approach to the Balance of Payments*. London: Allen and Unwin.

Frenkel, R. (2008). From the Boom in Capital Inflows to Financial Traps. In Ocampo, J. and Stiglitz, J., editors, *Capital Market Liberalization and Development*, pages 101–120. Oxford University Press: New York.

Garcia, M., Ayres, J.and Guillen, D., and Kehoe, P. (2018). The Monetary and Fiscal History of Brazil: 1960-2016. In Kehoe, P., Nicolini, J., and Sargent, T., editors, *The Monetary and Fiscal History of Latin America*. Becker Friedman Institute: Chicago, IL, USA.

Ghosh, A., Kim, J., Mendoza, E., Ostry, D. J., and Qureshi, M. (2013). Fiscal Fatigue, Fiscal Space and Debt Sustainability in Advanced Economies. *The Economic Journal*, 123(566):F4–F30.

Ghosh, A., Ostry, D. J., and Chamon, M. (2016). Two Targets, Two Instruments: Monetary and Exchange Rate Policies in Emerging Market Economies. *Journal of International Money and Finance*, 60:172–196.

Giavazzi, F. and Pagano, M. (1990). Can Severe Fiscal Contractions be Expansionary? Tales of Two Small European Countries. *NBER Macroeconomics Annual*, 5:75–111.

Godley, W. and Lavoie, M. (2007). Fiscal Policy in a Stock-Flow Consistent (SFC) Model. *Journal of Post Keynesian Economics*, 30(1):79–100.

Gopinath, G., Boz, E., Casas, C., Diez, F., and Gourinchas, P.-O. (2020). Dominant Currency Paradigm. *American Economic Review*, 110(3):677–719.

Greenidge, K., Craigwell, R., Thomas, C., and Drakes, L. (2012). Threshold Effects of Sovereign Debt: Evidence from the Caribbean. International Monetary Fund (IMF), Working Paper No. WP/12/157.

Guajardo, J., Leigh, D., and Pescatori, A. (2014). Expansionary Austerity? International Evidence. *Journal of the European Economic Association*, 12(4):949–968.

Guzman, M. and Heymann, D. (2015). The IMF Debt Sustainability Analysis: Issues and Problems. *Journal of Globalization and Development*, 6(2):387–404.

Hein, E. and Dodig, N. (2015). Finance-Dominated Capitalism, Distribution, Growth and Crisis – Long-Run Tendencies. In Hein, E., Detzer, D., and Dodig, N., editors, *The Demise of Finance-dominated Capitalism: Explaining the Financial and Economic Crises*. Edward Elgar Publishing.

Hein, E. and Stockhammer, E. (2010). Macroeconomic Policy Mix, Employment and Inflation in a Post-Keynsian Alternative to the New Consensus Model. *Review of Political Economy*, 22(3):317–354.

IMF (2011). Modernizing the Framework for Fiscal Policy and Public Debt Sustainability Analysis. International Monetary Fund (IMF), Paper prepared by the Fiscal Affairs Department and the Strategy, Policy, and Review Department.

IMF (2015). Evolving Monetary Policy Frameworks in Low-Income and Other Developing Countries. International Monetary Fund (IMF), Staff Report.

Jeanne, O. and Ranciere, R. (2009). The Optimal Level of International Reserves For Emerging Market Countries: A New Formula and Some Applications. *The Economic Journal*, 121(555):905–930.

Kalecki, M. (1943). The Burdent of the National Debt. *Bulletin of the Oxford University Institute of Economics and Statistics*, 5(5):76–80.

Keynes, J. M. (1936). *The General Theory of Employment, Interest and Money*. Palgrave Macmillan.

Khemraj, T. (2021). MMT-like Activities in a Managed Exchange Rate Economy. Unpublished.

Khemraj, T. and Pasha, S. (2012). Dual Nominal Anchors in the Caribbean. *Journal of Economic Studies*, 39(4):420–439.

Klein, M. and Pettis, M. (2020). *Trade Wars Are Class Wars: How Rising Inequality Distorts the Global Economy and Threatens International Peace*. Yale University Press.

Kohler, K. (2019). Exchange Rate Dynamics, Balance Sheet Effects and Capital Flows. A Minskyan Model of Emerging Market Boom-Bust Cycles. *Structural Change and Economic Dynamics*, 51:270–283.

Krugman, P. (1979). A Model of Balance of Payments Crises. *Journal of Money, Credit and Banking*, 11(3):311–325.

Krugman, P. (2020). The Case for Permanent Stimulus. In Baldwin, R. and Weder di Mauro, B., editors, *Mitigating the COVID Economic Crisis: Act Fast and Do Whatever It Takes*. CEPR Press.

Leigh, D., Igan, D., Simon, J., Topalova, P., Ros Karlsdottir, E., and Rozwadowski, F. (2012). Dealing with Household Debt. In *World Economic Outlook*, chapter 3. International Monetary Fund.

Lian, W., Presbitero, A., and Wiriadinata, U. (2020). Public Debt and r – g at Risk. International Monetary Fund (IMF), Working Paper No. WP/20/137.

Marcel, F. (2012). Capital Flows, Push versus Pull Factors and the Global Financial Crisis. *Journal of International Economics*, 88(2):341–356.

Mason, J. and Jayadev, A. (2018). A Comparison of Monetary and Fiscal Policy Interaction Under Sound and Functional Finance Regimes. *Metroeconomica*, 69(2):488–508.

Mauro, P., Romeu, R., Binder, A., and Zaman, A. (2015). A Modern History of Fiscal Prudence and Profligacy. *Journal of Monetary Economics*, 76:55–70.

Mauro, P. and Zhou, J. (2021). r - g < 0: Can We Sleep More Soundly? *IMF Economic Review*, 69(1):197–229.

Mian, A., Straub, L., and Sufi, A. (2021). A Goldilocks Theory of Fiscal Policy. https://scholar.harvard.edu/straub/publications/goldilocks-theory-fiscal-policy, Working Paper.

Mian, A. and Sufi, A. (2010). Household Leverage and the Recession of 2007-09. *IMF Economic Review*, 58(1):74–117.

Mishkin, F. (1999). Lessons from the Asian Crisis. *Journal of International Money and Finance*, 18(4):709–723.

Neary, P. and Stiglitz, J. (1983). Toward a Reconstruction of Keynesian Economics: Expectations and Constrained Equilibria. *Quarterly Journal of Economics*, 98:199–228.

Obstfeld, M., Shambaugh, J., and A., T. (2010). Financial Stability, the Trilemma, and International Reserves. *American Economic Journal: Macroeconomics*, 2(2):57–94.

Porcile, G. and Spinola, D. (2018). Natural, Effective and BOP-Constrained Rates of Growth: Adjustment Mechanisms and Closure Equations. *PSL Quarterly Review*, 71(285):139–160.

Prescott, E. (1986). Theory Ahead of Business-Cycle Measurement. *Carnegie-Rochester conference series on public policy*, 25(1):11–44.

Reinhart, C. and Reinhart, V. (2008). Capital Flow Bonanzas: An Encompassing View of the Past and Present. In Frankel, J. A. and Pissarides, C., editors, *NBER International Seminar on Macroeconomics*, pages 9–62. Univeristy of Chicago Press: Chicago.

Reinhart, C. and Rogoff, K. (2010). Growth in a Time of Debt. *American Economic Review: Papers and Proceedings*, 100(2):573–578.

Reinhart, C., Rogoff, K., and Savastano, M. (2003). Debt Intolerance. *Brookings Papers on Economic Activity, Economic Studies Program, The Brookings Institution*, 34(1):1–74.

Reis, R. (2021). The Constraint on Public Debt when r < g but g < m. Bank of International Settlement (BIS) Working Paper No. 939.

Ribeiro, R., McCauley, R., and Lima, G. (2017). Some Unpleasant Currency-Devaluation Arithmetic in a post Keynesian Macromodel. *Journal of Post Keynesian Economics*, 40(2):145–167.

Ribeiro, R., McCombie, J., and Lima, G. (2016). Exchange Rate, Income Distribution and Technical Change in a Balance of Payments Constrained Growth Model. *Review of Political Economy*, 28(4):545–565.

Sargent, T. and Wallace, N. (1981). Some Unpleasant Monetarist Arithmetic. *Federal Reserve Bank of Minneapolis Quarterly Review*, 5(3):1–17.

Schlicht, E. (2006). Public Debt as Private Wealth: Some Equilibrium Considerations. *Metroeconomica*, 57(4):494–520.

Serana, J. and Sousa, R. (2017). Does Exchange Rate Depreciation have Contractionary Effects on Firm-level Investment? Bank of International Settlement, BIS Working Paper No. 624.

Stewart, E. (2021). The risks of going too big on stimulus are real — but going too small could be riskier. https://www.vox.com/policy-and-politics/22268787/larry-summers-op-ed-biden-stimulus.

Storm, S. and Naastepad, C. W. M. (2015). Europe's Hunger Games: Income Distribution, Cost Competitiveness and Crisis. *Cambridge Journal of Economics*, 39(3):959–986.

Worrell, D. (1974). *The Theory of Optimal Foreign Exchange Reserves in a Developing Country: with Empirical Application to the Economy of Jamaica*. PhD thesis, McGill University.

Worrell, D., editor (2015). *Fiscal Sustainability and Debt in Small Open Economies – An Application to the Caribbean* . Caribbean Cenre for Money and Finance.

Yang, W., Fidrmuc, J., and Ghosh, S. (2015). Macroeconomic Effects of Fiscal Adjustment: A Tale of Two Approaches. *Journal of International Money and Finance*, 57:31–60.