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A Model of Macroprudential Frictions for Indirect Monetary Policy in The Bahamas

Sharon G. Branch, Jamere M. McIntosh and Allan S. Wright[1]

The Central Bank of The Bahamas

Abstract

The paper proposes a dynamic stochastic general equilibrium (DSGE) model of indirect monetary policy for The Bahamas. The model consists of '*macroprudential frictions*' including, a financial accelerator mechanism for the contract between entrepreneurs and financial intermediaries; a borrowing constraint for the contract between credit-constrained households and financial intermediaries; and 'distance-to-default' as a proxy for financial intermediaries' balance sheet strength. Data used to estimate the model is a mix of U.S. data—a proxy for shocks in the financial sector—and Bahamian macro aggregates. The results show that macroprudential shocks are substantial drivers of welfare for recessionary conditions, while standard productivity, monetary policy and investment-specific shocks drive welfare outside of recessions. Housing price dynamics, banking sector risk premiums, and discount factor shocks explain most of the variance decomposition of output and consumption. Comparisons of the model to one without macroprudential frictions reveal that ignoring relevant policy specifications leads to significant '*policy mistakes*'.

Keywords*: Macroprudential Frictions, Monetary Policy, Financial Stability*

JEL Codes*: E42, E52, F62*

[1] The views expressed in this paper are those of the authors and do not necessarily represent The Central Bank of The Bahamas. This paper should be considered a work in progress and as such, the authors would welcome any comments on the written text. SGBranch@centralbankbahamas.com; JMMcIntosh@centralbankbahamas.com; ASWright@centralbankbahamas.com

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Section 1. Introduction

The Bahamas' profile as a small-open, dependent economy, with a fixed exchange rate regime has inherently shaped the nature of monetary policy in the country, and consequently, the use of direct instruments. Therefore, this is a key contrast between The Bahamian economy and other small-open dependent economies, and larger economies, which have a different economic profile. Generally, central banks in larger economies, especially advanced economies, utilize and manipulate traditional monetary policy instruments (whether individually or in combination) for control of the money supply and/or the cost of funds in order to achieve intended economic outcomes. The standard direct monetary policy tools include those that operate by setting or limiting prices (interest rates) and quantities (money supply); such as the discount rate (i.e. interest rate controls), credit controls/ceilings, and directed lending. In contrast, key indirect monetary policy tools include those that act through market forces, such as the reserve requirement ratio, open market operations and central bank lending facilities (Alexander, Baliño and Enoch, 1996).

The standard monetary policy approaches of central banks in larger economies and advanced economies, however, have limited effectiveness for small-open-dependent economies, such as The Bahamas (Francis, 1986). The presence of a fixed exchange rate regime, which poses limitations for the independence of monetary policy, also influences the conduct of monetary policy and effectiveness of the standard instruments. The Central Bank of The Bahamas—mandated since 1974 to carry out independent monetary policy in The Bahamas—has operated with the objectives of fostering monetary and financial sector stability, economic growth and development and protecting the fixed exchange rate regime. With regard to the latter, support of the 1:1 parity between the Bahamian dollar and the U.S. dollar—which has existed since 1966—has also played an integral role in the Bank's monetary policy approach. The Central Bank of The Bahamas has statutory powers to implement standard monetary policy instruments, which it does, by using commonly available tools among central banks globally. These include the discount (bank) rate, the reserve requirement ratio, open market operations, and credit controls. The use of "moral suasion", identified by the Bank as an informal tool, supplements the formal ones.

The realities of the country's economic profile, and its fixed exchange rate regime, result in limited pass-through effects from the use of direct monetary policy instruments. In this context, this paper introduces a dynamic stochastic general equilibrium (DSGE) model in order to measure the impact of alternative policy specifications. The study examines whether replacing or supplementing monetary policy mechanisms with these tools will augment the effectiveness of monetary policy in The Bahamas. The model is a modification of the one introduced in Smets-Wouters (2003). Included are, macroprudential frictions such as a financial accelerator mechanism (Bernanke, 1999) for the contract between entrepreneurs and financial intermediaries, allowing for borrower leverage and an external finance premium. The second is a borrowing constraint, similar to the one by Kiyotaki and Moore

(1997) for the contract between credit-constrained households and financial intermediaries, imposed because of moral hazard problems preventing borrowers from financing beyond the liquidation value of collateral. Moreover the "distance-to-default" measure (Merton, 1974) is applied to the financial sector as a proxy for balance sheet strength. For a pre-crisis to post-crisis comparison, the model was evaluated over the 2007-2009 period in order to demonstrate the maximal impact of effective macroprudential policy. Specifically, this period began with a culmination of a time of robust economic performance, followed by the housing and financial market-induced Global Financial Crisis (GFC) and ensuing "Great Recession".

The remaining sections of this paper include a literature review of the empirical body of research, with particular focus on the models introduced by Smets & Wouters (2003), Bernanke, Gertler and Gilchrist (1999), Kiyotaki and Moore (1997), and Merton (1974). Further, Section 3 provides context of the limited pass-through effects of interest rates in The Bahamas, with several time series comparisons of various interest rate indicators, and a regression of the discount rate, and the main categories of commercial banks' lending rates. In Sections 4, the financial friction model is outlined, while Section 5 offers a description of the model and estimation results. Section 6 entail a discussion of the findings, along with implications for policy and the concluding portion, which includes areas for further research.

Section 2. Review of Literature

Varied models have been developed which estimate the sources of business cycle movements and analyse the impacts of shocks on output. The one proposed in this paper is a modification of that introduced by Smets and Wouters (2003). The Smets-Wouters (SW) model entails key features of DSGE models relevant to the traditional Keynesian ones. Additional frictions, as well as structural shocks, were introduced to the model. These allowed for parameter estimation via Bayesian techniques and an analysis of the sources of movements in the business cycle. Empirical estimates revealed a considerable degree in price and age stickiness in the euro area, which was also useful for the analysis of monetary policy. The effects of two types of monetary shocks: temporary and persistent, were also measured, as were a number of non-monetary shocks.

Another frequently included element in these DSGE models are contracts between economic agents (particularly consumers and firms), and financial intermediaries. Building on developments in the economics of imperfect information from the 1970s, Bernanke, Gertler and Gilchrist (1996) examine the financial accelerator—the concept that changes in credit market conditions amplify and propagate macroeconomic shocks. The proposed model includes a partial equilibrium for the lender (principal)/borrower (agent) relationship, and a general equilibrium for macroeconomic dynamics that incorporated the financial accelerator mechanism into a model of the business cycle. Empirical findings, which were drawn from a panel of manufacturing firms, indicated that smaller firms showed more procyclical variation in inventories and short-term debt than do larger firms, consistent with the hypothesis that consumers, small firms and firms with weak balance sheets bear the brunt of economic downturns, and therefore should receive reduced access to credit, relative to other types of borrowers, following economic shocks.

In a subsequent study, Bernanke et al (1999) also developed a DSGE model for the role of credit market frictions in business cycle fluctuations. Similar to the 1996 study, a financial accelerator is included. However, for the 1999 study, features were added to the financial accelerator to augment its empirical relevance. These included money and price stickiness to measure the impact of credit market frictions on monetary policy transition, and a decision lag for investment to allow for a lead-lag relationship between asset prices and investment and to generate hump-shaped output dynamics. Similar to the model in the 1996 study, a partial equilibrium component was embedded into the generalized model (in the case of the 1999 study, the lender-entrepreneur relationship) to allow for endogeneity of the safe interest rate, capital return and relative price of capital.

Kiyotaki and Moore (1997) more closely examine the enforceability of debt contracts vis-à-vis Bernanke et al (1996). The latter study affirmed the welfare of consumers and small firms being highly susceptible to macroeconomic shocks, noting that borrowers with higher agency costs should have a lower share of credit extended to them at the onset of recessions. By comparison, Kiyotaki and Moore (1997) incorporate financial frictions via the limited enforceability approach which holds for the lack of perfect enforceability of debt contracts, limited recovery for the lender in the case of default, and the imposing of credit restrictions for the same. Therefore, for this model, durable/collateralizable assets play a dual role as factors of production and collateral on loans.

To quantify the balance sheet strength of firms, the model proposed in this paper employs the distance-to-default as a proxy measure, which allows for an assessment of the firm's credit risk. This measure is based on the structural default model of Merton (1974)—also referred to as the "Merton Model"—in which equity is treated as a European call option. By back-solving the Black-Scholes Options pricing formula, the Merton Model derives the firm's implied market value and volatility (Shah, Singh and Aggarwal (2023)). In later studies, Vasicek (1984) and Crosbie and Bohn (2003) extended the Merton Model. However, the model proposed by Vasicek (1984) diverges from earlier methods of credit analysis, with a particular focus given to market (information) efficiency. Similar to Merton, they assume the firm's equity to be an option, with a key extension being cash payouts, including dividends made in the event of default. Crosbie and Bohn (2003) proposed a model for default risk by the Moody's KMV Company ("MKMV Model"). The model provides a measure of "Expected Default Frequency (EDF)", the probability of default for a publicly traded company during the forthcoming year. Distance-to-default is one component of the model, along with estimated asset value-and-volatility, and default probability.

A number of further studies have involved extensions of the model proposed by Bernanke et al (1999) to explain the role of the financial conditions in the business cycle. Gertler, Gilchrist and Natalucci (2005) developed a model of a small open economy which examined the linkage between exchange rate regime and financial distress. The model was calibrated to reflect the behaviour of the Korean economy during the 1997-1998 financial crisis period in the country. In addition to extensions from the Bernanke et al (1999), the model was modified to include a measure for changes in productivity, and link borrower balance sheets to demand for capital. Shocks were applied to illustrate how the exchange rate regime (flexible, fixed or hybrid) might exacerbate welfare losses. Similar to Gertler et al (2005), Lee and Rhee (2013) developed a model that included an extension of the model of Bernanke et al (1999) which included financial factors. The main modification to Bernanke et al was the proposal of a two-country economy, one being the small open economy. Also similar to Gertler et al (2005) was the study's use of the Korean economy to estimate the DSGE model. However, the model used by Lee and Rhee (2013) was estimated using Bayesian methods, as was done in a later study by Smets and Wouters (2007) and Adolfson, Laséen, Lindé and Villani (2007). Kitano and Takaku (2018) incorporated the financial accelerator mechanism of Bernanke et al (1999) in their model of a small open economy, the structure of which is consistent with the one developed in studies by Galí and Monacelli (2005) and Faia and Monacelli (2008). Similar to Gertler et al (2005) Kitano and Takaku (2018) analysed the welfare impacts of monetary policy with respect to exchange rate regime. The findings indicate superiority of the flexible exchange rate for the economy without the financial accelerator and superiority of the fixed exchange rate for the economy with the financial accelerator.

Section 3. The Pass-Through Effects of Interest Rates: The Case of The Bahamas

The discount (bank) rate is the key monetary policy rate for The Bahamas and is linked to the commercial banks' prime lending rate. Adjustments in the discount rate by the central bank are followed by corresponding changes in the prime rate. Within the last twenty years, there have been three downward adjustments to the discount rate (February 2005, June 2011 and December 2016). The 2005 adjustment was prompted by persistent levels of excess liquidity, while the 2011 and 2016 lowering of the rate was attributed to providing support for positive growth outlook. An analysis of trends in interest rates in The Bahamas over the past twenty years showed low pass-through effects when examining movements in the discount rate and other rates.

Table 1 shows regression results for the discount rate (independent variable) and the weighted average lending rate (dependent variables). Based on the results, the higher the adjusted R-squared statistic, the stronger the pass through effect from the policy rate to the lending rate. According to the results for the weighted average lending rate (differenced) on the discount rate, an adjusted R-squared of 0.172, indicated the weak pass through effect of a change in the policy rate. Hence, this is an indication that the standard monetary policy for

The Bahamas has little to no effect. Therefore, signalling that macroprudential policies need to complement the standard monetary policy to be effective in the market.

Table 1: Regression Output
(Differenced Weighted Average Lending Rate on Discount Rate)

Dep. Variable:	Weighted average lending rate	**R-squared:**	0.180
Model:	OLS	**Adj. R-squared:**	0.172
Method:	Least Squares	**F-statistic:**	22.39
Date:	Fri, 25 Mar 2022	**Prob (F-statistic):**	7.17e-06
Time:	10:28:16	**Log-Likelihood:**	-119.92
No. Observations:	104	**AIC:**	243.8
Df Residuals:	102	**BIC:**	249.1
Df Model:	1		

	coef	**std err**	**t**	**P> |t|**	**[0.025**	**0.975]**
const	9.0577	0.484	18.717	0.000	8.098	10.018
Discount Rate	0.4427	0.094	4.732	0.000	0.257	0.628

Omnibus:	1.388	**Durbin-Watson:**	0.354
Prob(Omnibus):	0.499	**Jarque-Bera (JB):**	1.273
Skew:	0.128	**Prob(JB):**	0.529
Kurtosis:	2.522	**Cond. No.**	34.2

Notes:
[1] First Difference of Weighted average lending rate and Discount Rate are Cointegrated at a level of 1% significance, p-value: 0.4765.

Section 4. The Financial Friction Model

A financial friction in the business sector via the accelerator mechanism was introduced in the model. More specifically, there are two additional economic agents involved in the capital investment process, entrepreneurs and capital-goods producers. Entrepreneurs effectively choose the capital stock each period. Capital investment is financed by external borrowing and net worth. Net worth of the entrepreneurs is defined as the retained earnings from the previous period.

The key equation that characterizes the financial accelerator mechanism is given as:

$$E_t\left(\frac{R_{t+1}^K}{R_t^f}\right) = f\left(\frac{Q_t K_t^p}{NW_t}\right)\varepsilon_t^{rp} \qquad \text{eq (1)}$$

where K_t^p is the physical capital stock, Q_t is the price of capital, NW_t is the net worth of the entrepreneur, ε_t^{rp} is a shock to the risk premium and f is assumed to be an increasing function.

The equation shows that the external finance premium, defined by the ratio of expected return on capital to the intermediary's funding rate $\left(E_t R_{t+1}^K / R_t^f\right)$, will be an increasing function of the ratio of total assets over net worth $\left(Q_t K_t^p / NW_t\right)$.

The return on capital is determined by the marginal productivity of capital and the price change of capital:

$$R_t^k = \frac{MP_K + (1-\delta)Q_t}{Q_{t-1}} \qquad \text{eq (2)}$$

The entrepreneurs' net worth is defined by net returns after repaying the debt obligation. The law of motion for the net worth is thus given by:

$$NW_t = \vartheta\left[R_t^k Q_{t-1} K_{t-1}^p - R_{t-1}^f\left(Q_{t-1} K_{t-1}^p - NW_{t-1}\right)\right]\varepsilon_t^{nw} \qquad \text{eq (3)}$$

where ϑ is the survival rate of the entrepreneurs for each period. Equation (3) shows that the net worth of the surviving entrepreneurs is the retained earnings from the investment after subtracting off the portion claimed by the intermediary. ε_t^{nw} is a shock to the entrepreneurs' net worth, which represents the unexpected gain or loss that affects the entrepreneur's balance sheet.

Given the size of physical capital stock, entrepreneurs also determine the utilization rate. It is assumed that capital utilization is costly with costs determined by $a(u_t)$, and the entrepreneurs' decision regarding capital utilization is made by solving the following optimization problem [1]:

$$\max_u MP_k \cdot K_t - a(u_t)K_{t-1}^p,\ K_t = K_{t-1}^p \cdot u_t \qquad \text{eq (4)}$$

Capital goods producers purchase I_t amounts of consumption goods at a price of one, and turn them into the same amount of new capital. Transformation costs, $s(\cdot)$, arise during the process, and the capital is resold to entrepreneurs at price Q_t. Capital goods producers maximize future discounted expected return, given by the following optimization problem:

$$\max_I \sum_{s=0}^{\infty} \bar{\beta}^s \varepsilon_{t+s}^{\beta} E_t\left[Q_{t+s} - \left\{1 + s\{I_{t+s}/I_{t+s-1}\}\varepsilon_{t+s}^i\right]I_{t+s}\right. \qquad \text{eq (5)}$$

where ε_t^i is the investment specific shock that affects the efficiency of capital accumulation process.

[1]The first-order condition is given by $MP_k = a'(u_t)$, which equates marginal benefit and marginal cost.

In terms of credit constrained borrowing households, they are distinguished by patient and impatient households. Impatient households have lower future discount parameters than patient households ($\beta' < \beta$). There are a continuum of agents in each household group. The economic size of each group is determined by its share of wage income, which is characterized by the parameter μ. Impatient households are borrowers in the steady state and around its neighborhood. Households have preferences over not only consumption goods, but also housing goods.

Patient households maximize

$$E_t \sum_{s=0}^{\infty} \beta^s \varepsilon_{t+s}^{\beta} \left[\frac{1}{1-\sigma_c} J_{t+s}^{1-\sigma_c}\right] \exp\left[\frac{\sigma_c - 1}{1+\sigma_l} L_{t+s}^{1+\sigma_l}\right] \qquad \text{eq (6)}$$

where J is a composite of consumption and housing goods:

$$J_t = \left[(1-\psi)(C_t - \lambda C_{t-1})^{1-\sigma_h} + \psi \varepsilon_t^{\psi} (H_t)^{1-\sigma_h}\right]^{\frac{1}{1-\sigma_h}} \quad \text{eq (7)}$$

subject to the budget constraint:

$$\begin{aligned} & C_{t+s} + \frac{B_{t+s}}{R_{t+s}P_{t+s}} + T_{t+s} + \frac{Q_{t+s}^h}{P_{t+s}} H_{t+s} \\ \leq \; & \frac{B_{t+s}}{P_{t+s}} + \frac{W_{t+s}^h L_{t+s}}{P_{t+s}} + \frac{Q_{t+s}^h}{P_{t+s}} (1-\delta_h) H_{t+s-1} + Div_{t+s} \end{aligned} \quad \text{eq (8)}$$

In the utility function, C is consumption, H is housing goods and L is the labor supply. In the budget constraint, B is the nominal deposit, R_t is the nominal gross saving interest rate, P is the price of consumption goods, T is lump-sum tax, Q^h is nominal housing price and δ_h is the depreciation rate of housing goods. W^h is the wage received and Div is the dividend income from firms. ε_t^{ψ} is a preference shock for the housing goods that affects housing demand. ε_t^{β} is a shock affecting the discount factor, which is different from the financial friction shock in the standard SW model. This is because the discount factor shock only affects the intertemporal consumption decision while financial friction shock in the SW model affects both the intertemporal consumption and investment decision as it introduces a wedge between the rate at which households save and borrow.

Impatient households maximize:

$$E_t \sum_{s=0}^{\infty} \beta'^s \varepsilon_{t+s}^{\beta} \left[\frac{1}{1-\sigma_c} J_{t+s}'^{\;1-\sigma_c}\right] \exp\left[\frac{\sigma_c - 1}{1+\sigma_l} L_{t+s}'^{\;1+\sigma_l}\right] \quad \text{eq (9)}$$

where

$$J_t' = \left[(1-\psi)(C_t' - \lambda C_{t-1}')^{1-\sigma_h} + \psi \varepsilon_t^{\psi} H_t'^{1-\sigma_h}\right]^{\frac{1}{1-\sigma_h}} \quad \text{eq (10)}$$

subject to the budget constraint:

$$\begin{aligned} & C_{t+s}' + T_{t+s}' + \frac{Q_{t+s}^h}{P_{t+s}} H_{t+s}' + \frac{B_{t+s-1}' R_{t+s-1}^b}{P_{t+s}} \\ \leq \; & \frac{B_{t+s}'}{P_{t+s}} + \frac{W_{t+s}^{h\prime} L_{t+s}'}{P_{t+s}} + \frac{Q_{t+s}^h}{P_{t+s}} (1-\delta_h) H_{t+s-1}' + Div_{t+s}' \end{aligned} \quad \text{eq (11)}$$

and the collateral (loan-to-value) constraint:

$$\frac{B_t'}{P_t} \leq m \varepsilon_t^{dbt} E_t q_{t+1}^h H_t' \quad \text{eq (12)}$$

The parameter m determines the steady state loan-to-value (LTV) ratio, which is the ratio of debt to collateral value. Noteworthy is that, the impatient households' ability to borrow is limited by the value of collateral assets that can be liquidated. Housing goods are used as the collateral asset, and the constraint binds around the steady-state and its neighbourhood. The LTV ratio is assumed to vary over time, as ε_t^{dbt} denotes an external disturbance to lending standards.

The financial sector was modelled by focusing on the relationship between intermediaries' balance sheet and their ability to intermediate credit. This type of friction is believed to be a key factor during the 2007-2009 financial crisis.

One of the novel aspects of the modelling approach used in this study is the 'distance to default' as an observable variable to capture the riskiness of the financial sector. Distance to default is an indicator of default probability and demonstrates how equity could be modelled as a call option on the assets of the firm with a strike price equal to the firm's liabilities. By assuming a simple capital structure, Merton (1974) was able to calculate the default probability via the derivative pricing equations.

Specifically, assume the firm's assets are financed by equity issued at time t denoted byS_t, and zero-coupon debt issued at $t(D_t)$ with a face value of F and maturity date M. The market value of the firm at any date t is given by the sum of the market value of debt and equity. Therefore, the accounting identity $V_t = S_t + D_t$, where V_t denotes firm value, holds for each period. Under these assumptions, the bondholders are entitled to a time- M cash flow of $\min[V_M, F]$ and since equity holders are the residual claimants, the value of equity at time M is given by $\max[V_M - F, 0]$. At any time $t < M$, the value of these derivative securities is:

$$\begin{aligned} S_t &= e^{-r(M-t)}\mathbb{E}_t^Q\{\max[V_M - F, 0]\} \\ D_t &= e^{-r(M-t)}\mathbb{E}_t^Q\{\min[V_M, F]\} \end{aligned} \quad \text{eq (13)}$$

where the expectation is taken with respect to the risk-neutral probability measure and the risk-free rate r is assumed to be constant over time.

Assuming a geometric Brownian motion for firm value, Merton (1974) showed the probability of default for the firm can be backed out of equations (16) and (17), and is given by:

$$\pi_t^D = \Pr\left(-\frac{\ln(V_t) - \ln(F) + (\mu_V - \sigma_V^2/2)J}{\sigma_V\sqrt{M}} \geq \varepsilon_{t+M}\right) \quad \text{eq (14)}$$

where ε_{t+M} is white noise.

The distance to default can then be defined as:

$$DD_t = \frac{\ln(V_t/F) + (\mu_V - \sigma_V^2/2)M}{\sigma_V\sqrt{M}} \quad \text{eq (15)}$$

Default occurs when the ratio of firm value to debt (V_t/F) drops below unity or the log of the ratio is negative. The distance to default DD_t can be interpreted as a z-score, which gives the number of standard deviations that the log of this ratio needs to deviate from its mean in order for default to occur. In other words, the probability of bankruptcy depends upon the distance between the current value of the firm's assets and the face value of its liabilities, adjusted for the expected growth in asset value relative to asset volatility.

Section 5. Model Specification and Results

5.1 Model Description and Estimation

In this study, a dynamic stochastic general equilibrium (DSGE) model of indirect monetary policy for The Bahamas was developed, with substantial “macroprudential frictions”, which measures the impact of alternative policy specifications, such as changes to leverage ratios. The rationale for the model is that traditional monetary policy approaches, such as changes in the interest rate, have weak pass-through effects on the economy in The Bahamas due that the fact it is a fixed exchange rate economy, pegged to the United States dollar, and has limited monetary policy tools.

The model is a modification of the standard Smets-Wouters (2003) set, which includes macroprudential frictions such as, a financial accelerator mechanism of Bernanke (1999) that applies to the contract between entrepreneurs and the financial intermediaries, allowing for borrower leverage and an external finance premium. In addition, the model comprised a borrowing constraint, similar to Kiyotaki (1997) which applies to the contract between credit-constrained households and the financial intermediaries. The constraint is imposed because of moral hazard problems that prevent borrowers from financing beyond the liquidation value of the collateral. Further, the model introduces households who are credit-constrained because they are impatient, to use their housing goods as collateral, allowing housing market conditions to impact the business cycle. Finally, balance sheets of financial intermediaries are allowed to affect their ability to draw loanable funds and therefore to intermediate credit. Specifically, the ‘distance-to-default’ measure of Merton (1974), applied to the financial sector is used as a proxy for balance sheet strength.

The model is estimated using Bayesian methods with a mixture of U.S. and Bahamian data. The U.S. data is used to estimate the financial frictions (interest rate spread, housing prices, bank leverage), with an emphasis on the 2007-09 period. The extent to which these financial shocks penetrated smaller open economies is then estimated through the use of Bahamian macro aggregate variables (consumption, investment, GDP deflator, 10-year Bahamian interest rate) from the period of 1996-2022. Our justification for using U.S. data to measure financial frictions in The Bahamas comes from the correlation between small open economies and US-generated shocks (Cormun and De Leo (2020)).

We calibrate some parameters consistent with Smets-Wouters (2007). The depreciation rate of housing, δh, is calibrated as 0.1, greater than the depreciation rate of non-residential capital. The parameter ψ represents the weight on housing in the utility function and is chosen at 0.15. These two calibrated parameters pin down the steady-state residential investment-nonresidential investment ratio at approximately 4:1. The parameter μ is the labor income share of saving household, which is set at 0.75, the steady-state loan-to-value ratio, *m*, is the ratio of the borrowing household and is chosen to be 0.75, consistent with US data.

The remaining parameters are estimated using Bayesian analysis. The priors are taken from Suh and Walker (2016) and are relatively standard in the literature. Tables 2 and 3 provide the prior and posterior results. The posterior values in parentheses come from using only U.S. data; that is, we assume that only U.S. data are used for the full estimation. The non-parentheses values represent Bahamian macro aggregates. Table 2 shows that the median and mean values for parameter estimates are roughly consistent across the countries, while the 5-95 percentiles are quite different. This is not surprising given that the model is linearized around a common steady state and given the correlation in U.S. and Bahamian data. Similarly, Table 3 shows little difference between US and Bahamian data in shock estimation.

Table 2: Prior and Posterior Distribution of Structural Parameters

		Prior Distribution			Posterior Distribution			
		Distr.	Mean	St.Dev.	Median	Mean	5 pct	95 pct
φ	Non-residential capital adjustment cost	Normal	4.00	1.50	5.64(6.13)	5.87(6.16)	4.05(4.37)	7.95(7.91)
σ_c	Elasticity of intertemporal substitution	Normal	1.50	0.37	1.30(1.31)	1.33(1.32)	1.05(1.11)	2.23(1.54)
λ	Habit formation	Beta	0.70	0.10	0.87(0.71)	0.87(0.71)	0.56(0.66)	0.92(0.77)
ξ_w	Wage rigidity	Beta	0.50	0.10	0.90(0.90)	0.90(0.90)	0.84(0.86)	0.96(0.94)
σ_l	Labor elasticity	Normal	2.00	0.75	2.52(2.14)	2.52(2.16)	1.18(1.23)	3.15(3.07)
ξ_p	Price rigidity	Beta	0.50	0.10	0.65(0.76)	0.65(0.76)	0.59(0.69)	0.75(0.83)
ι_w	Wage indexation	Beta	0.50	0.15	0.54(0.55)	0.54(0.55)	0.32(0.34)	0.75(0.77)
ι_p	Price indexation	Beta	0.50	0.15	0.35(0.24)	0.36(0.25)	0.10(0.10)	0.56(0.39)
Ψ	Capital Utilization	Beta	0.50	0.15	0.70(0.69)	0.69(0.68)	0.45(0.51)	0.88(0.85)
Φ	Fixed cost in production	Normal	1.25	0.12	1.54(1.54)	1.55(1.54)	1.35(1.42)	1.75(1.67)
r_π	MP reaction to inflation	Normal	1.50	0.25	1.53(1.52)	1.54(1.52)	1.05(1.36)	1.85(1.69)
ρ	MP rigidity	Beta	0.75	0.10	0.80(0.79)	0.80(0.79)	0.76(0.75)	0.83(0.83)
r_y	MP reaction to output gap	Normal	0.12	0.05	-0.01(-0.01)	-0.01(-0.01)	-0.04(-0.02)	0.00(0.00)
r_{Δ_y}	MP reaction to output gap change	Normal	0.12	0.05	0.10(0.15)	0.10(0.15)	0.08(0.11)	0.25(0.18)
$\bar{\pi}$	Steady-state inflation	Gamma	0.62	0.10	0.83(0.83)	0.83(0.83)	0.67(0.66)	0.99(1.00)
$\bar{\beta}$	Steady-state discount rate	Gamma	0.25	0.10	0.27(0.29)	0.29(0.30)	0.15(0.15)	0.44(0.46)
$\bar{l}$	Steady-state hours worked	Normal	0.00	2.00	-2.37(-2.32)	-2.37(-2.34)	-5.65(-4.46)	-0.01(-0.14)
$\bar{\gamma}$	Steady-state trend growth rate	Normal	0.40	0.10	0.25(0.40)	0.25(0.40)	0.02(0.36)	0.54(0.45)
α	Capital share in production	Normal	0.30	0.05	0.12(0.17)	0.12(0.17)	0.08(0.14)	0.29(0.20)
φ^h	Residential capital adjustment cost	Normal	0.30	0.05	0.25(0.30)	0.25(0.30)	0.20(0.25)	0.39(0.35)
σ^h	Elasticity, consumption and housing	Normal	1.50	0.37	1.23(1.23)	1.25(1.24)	1.08(1.07)	1.41(1.40)
$\bar{\pi}^{qh}$	Steady-state housing price inflation	Normal	0.2	0.15	0.20(0.20)	0.20(0.20)	0.11(0.11)	0.30(0.30)
χ^e	Financial accelerator	Normal	0.05(0.2)	0.02(0.05)	0.004(0.004)	0.004(0.004)	-0.002(-0.002)	0.009(0.009)
χ^f	Interbank rate elasticity to DTD	Normal	-0.05(-0.2)	0.02(0.05)	-0.01(-0.02)	-0.01(-0.02)	-0.04(-0.06)	0.02(0.02)
$\chi^{DTD,K}$	DTD elasticity to return on capital	Normal	0.05(0.2)	0.02(0.05)	0.02(0.02)	0.02(0.02)	0.00(0.00)	0.03(0.03)
$\chi^{DTD,H}$	DTD elasticity to housing price	Normal	0.05(0.2)	0.02(0.05)	0.04(0.05)	0.04(0.05)	0.02(0.03)	0.06(0.07)

Table 3: Prior and Posterior Distribution of Shock Processes

		Prior Distribution			Posterior Distribution			
		Distr.	Mean	St.Dev.	Median	Mean	5 pct	95 pct
σ_a	SE, productivity	Invgam	0.10	2.00	0.42(0.42)	0.42(0.42)	0.38(0.38)	0.46(0.46)
σ_b	SE, discount factor	Invgam	0.10	2.00	0.35(0.35)	0.35(0.35)	0.30(0.30)	0.40(0.40)
σ_g	SE, government	Invgam	0.10	2.00	0.46(0.46)	0.46(0.46)	0.41(0.41)	0.50(0.50)
σ_I	SE, investment	Invgam	0.10	2.00	0.34(0.34)	0.34(0.34)	0.27(0.27)	0.40(0.40)
σ_r	SE, monetary	Invgam	0.10	2.00	0.22(0.23)	0.23(0.23)	0.20(0.20)	0.25(0.25)
σ_p	SE, inflation markup	Invgam	0.10	2.00	0.13(0.13)	0.13(0.13)	0.11(0.11)	0.16(0.16)
σ_w	SE, wage markup	Invgam	0.10	2.00	0.29(0.29)	0.29(0.29)	0.25(0.25)	0.33(0.33)
σ_{irs}	SE, interbank spread	Invgam	0.10	2.00	0.06(0.06)	0.06(0.06)	0.06(0.06)	0.07(0.07)
σ_{dtd}	SE, distance to default	Invgam	0.10	2.00	0.19(0.18)	0.19(0.19)	0.17(0.17)	0.21(0.20)
σ_{rp}	SE, risk premium	Invgam	0.10	2.00	0.09(0.09)	0.09(0.09)	0.08(0.08)	0.10(0.10)
σ_{nw}	SE, net worth	Normal	3.00	0.50	2.10(2.11)	2.11(2.12)	1.89(1.89)	2.33(2.34)
σ_{ah}	SE, housing investment	Normal	3.00	0.50	2.07(2.07)	2.08(2.08)	1.84(1.84)	2.32(2.30)
σ_h	SE, housing demand	Normal	5.00	1.00	5.19(5.18)	5.20(5.20)	4.54(4.53)	5.86(5.86)
ρ_a	AR(1), productivity	Beta	0.50	0.20	0.97(0.97)	0.97(0.97)	0.96(0.96)	0.99(0.99)
ρ_b	AR(1), discount factor	Beta	0.50	0.20	0.39(0.39)	0.40(0.39)	0.22(0.22)	0.57(0.56)
ρ_g	AR(1), government	Beta	0.50	0.20	0.97(0.97)	0.96(0.96)	0.94(0.94)	0.99(0.99)
ρ_I	AR(1), investment	Beta	0.50	0.20	0.83(0.83)	0.83(0.83)	0.78(0.77)	0.89(0.88)
ρ_r	AR(1), monetary	Beta	0.50	0.20	0.13(0.13)	0.13(0.13)	0.04(0.03)	0.22(0.23)
ρ_p	AR(1), inflation markup	Beta	0.50	0.20	0.92(0.92)	0.91(0.92)	0.86(0.86)	0.98(0.98)
ρ_w	AR(1), wage markup	Beta	0.50	0.20	0.88(0.86)	0.88(0.81)	0.61(0.61)	0.97(0.97)
ρ_{irs}	AR(1), interbank spread	Beta	0.50	0.20	0.70(0.74)	0.70(0.73)	0.58(0.60)	0.82(0.88)
ρ_{dtd}	AR(1), distance to default	Beta	0.50	0.20	0.98(0.98)	0.98(0.98)	0.96(0.96)	0.99(0.99)
ρ_{rp}	AR(1), risk premium	Beta	0.50	0.20	0.96(0.96)	0.96(0.96)	0.94(0.95)	0.98(0.98)
ρ_{nw}	AR(1), net worth	Beta	0.50	0.20	0.43(0.43)	0.44(0.44)	0.27(0.28)	0.60(0.60)
ρ_{ah}	AR(1), housing investment	Beta	0.50	0.20	0.97(0.97)	0.97(0.97)	0.95(0.95)	0.99(0.99)
ρ_h	AR(1), housing demand	Beta	0.50	0.20	0.97(0.97)	0.97(0.97)	0.95(0.95)	0.99(0.99)
μ_p	MA(1), inflation markup	Beta	0.50	0.20	0.80(0.81)	0.79(0.80)	0.68(0.69)	0.90(0.90)
μ_w	MA(1), wage markup	Beta	0.50	0.20	0.81(0.81)	0.77(0.76)	0.54(0.50)	0.95(0.96)
ρ_{gy}	Government spending correlation	Beta	0.50	0.20	0.49(0.49)	0.50(0.49)	0.34(0.35)	0.64(0.64)

Note: The value in parentheses is using US data only.

5.2 Analysis of Results

The main takeaway of the paper is that macroprudential factors are important for understanding Bahamian macro aggregates. Macroprudential shocks, such as shocks to the discount factor, housing market and financial sector, are substantial drivers of welfare when the shock variances are calibrated to recessionary levels. It also suggested that outside of recessions, the standard shocks (e.g., productivity, monetary policy, investment specific shocks) drive the bulk of welfare. House price dynamics, banking sector risk premium and discount factor shocks also explain a majority of the variance decomposition of output and consumption. In comparing a macroprudential model vis-à-vis a model without macroprudential frictions, it was found that ignoring leverage ratios, risk premium and the housing market lead to significant policy mistakes by policy makers. Specifically, the monetary authority is likely to over-tighten monetary policy—such as changes in leverage ratios, which can have adverse effect on consumption and output—versus macroprudential changes.

Figure 1: Historical Decomposition of Consumption

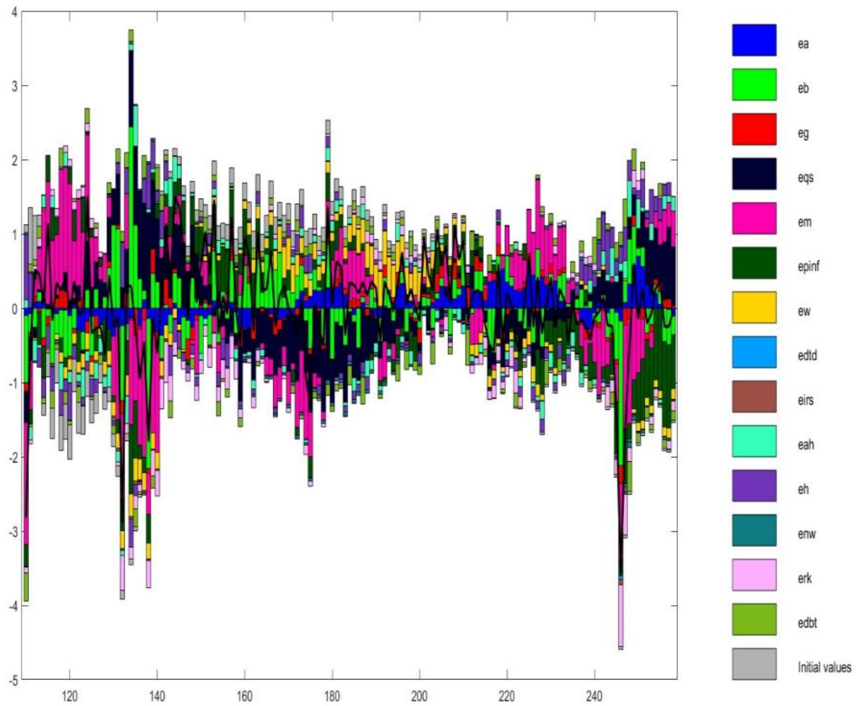
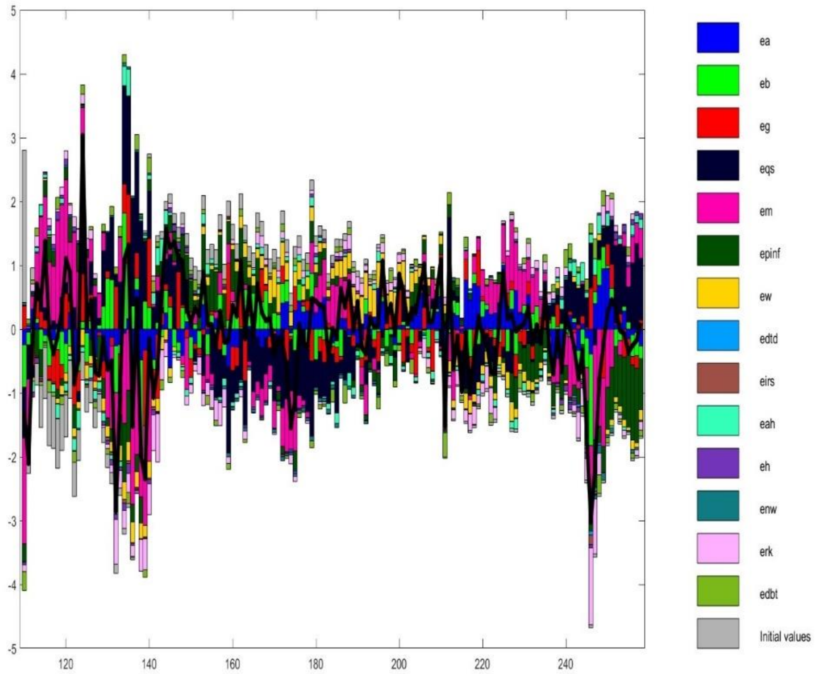


Figure 1 is a historical variance decomposition of consumption. It provides evidence of these facts by demonstrating that discount shocks are most important for explaining variation in consumption, especially during recessions or times of negative consumption growth. Specifically, the figure demonstrates that the lion's share of variation in consumption can be attributed to discount factor shocks (eb), followed by shocks to the risk premium. Shocks to the interest rate also play an important role (em), especially during times of contraction.

Not surprisingly, shocks to productivity and investment, drive the historical decomposition of output (see Figure 2). However, during substantial drops, the discount factor shock again plays a significant role. This is due to the fact that consumption contributes to output demand and a substantial slowdown in consumption parlays into a drop in output the following quarter.

Figure 2: Historical Decomposition of Output



The economic intuition behind these financial frictions is as follows. In examining financial friction, an analysis of a standard deviation shock to the risk premium leads to a rise in the entrepreneur risk, as debt levels (entrepreneur leverage) deteriorate and the businessperson moves closer to a default level. In terms of the entrepreneur net worth, a one standard deviation shock will result in a decrease in the net worth of the business person, as the risk spread and the leverage ratio reduce, as more entrepreneurial assets will have to be expended on servicing their debt and lowering the distance to default ratio. Further, a one standard deviation shock to bank spread is likely to result in a narrowing in spreads, as banks become more profitable, as the entrepreneur service levels increase. With regard to the distance to default, a shock will lead to a widening in the bank spread for the entrepreneur and their distance of default. For all shocks originating outside of the financial sector, bank spread and the default level will be mainly impacted, with a widening in spreads and entrepreneurs more likely to default.

Table 4: Variance Decomposition, Financial Variables (%)

Variables / Shocks	Entrepreneur risk spread	Entrepreneur leverage	Interbank spread	Distance to default
Risk premium	90.5	25.1	0.9	2.7
Entrepreneur net worth	5.6	43.4	0.7	0.1
Interbank spread	2.2	2.3	96.8	0.2
Distance to default	1.2	0.9	1.0	84.6
Non-financial	0.5	28.3	0.6	12.5

Table 5: Variance Decomposition, Non-Financial Variables (%)

Variables / Shocks	ΔOutput	ΔConsumption	ΔInvestment	ΔInflation	ΔHousing Price	ΔHousing Investment
Productivity	5.7	3.7	1.8	9.7	0.2	1.0
Discount factor	11.6	35.0	0.2	0.5	1.2	1.0
Gvt. spending	10.9	1.4	0.0	1.3	0.3	0.5
Inv. specific	5.8	13.0	65.1	7.1	0.3	0.9
Monetary	11.7	14.8	5.1	2.5	8.1	14.2
Housing demand	15.2	1.4	0.8	18.8	30.0	64.9
Housing supply	10.5	1.0	0.1	0.2	58.8	12.0
Risk premium	3.9	1.8	10.2	6.8	0.1	0.2
Other financial	0.2	0.1	0.5	0.6	0.0	0.0

Variance decompositions show how much a shock contributes to the forecast error variance of each variable. We use variance decomposition to understand the importance of each financial friction channel in the model. Tables 4 and 5 present the variance decomposition of financial and non-financial variables at the posterior mean. Table 4 shows that financial shocks (risk premium shock, net worth shock, interbank spread shock, and distance to default shock) explain a large part of the forecast error variance of financial indicator variables. Table 5 shows that the discount factor shock plays the largest role in explaining the change in consumption. Investment specific shocks are most important for understanding business cycle dynamics as they account for 26.3% of output and 65% of investment. Housing services play a crucial role in explaining the variance of output.

Figure 3. Impulse Response of Consumption to Discount Rate and Monetary Policy Shock

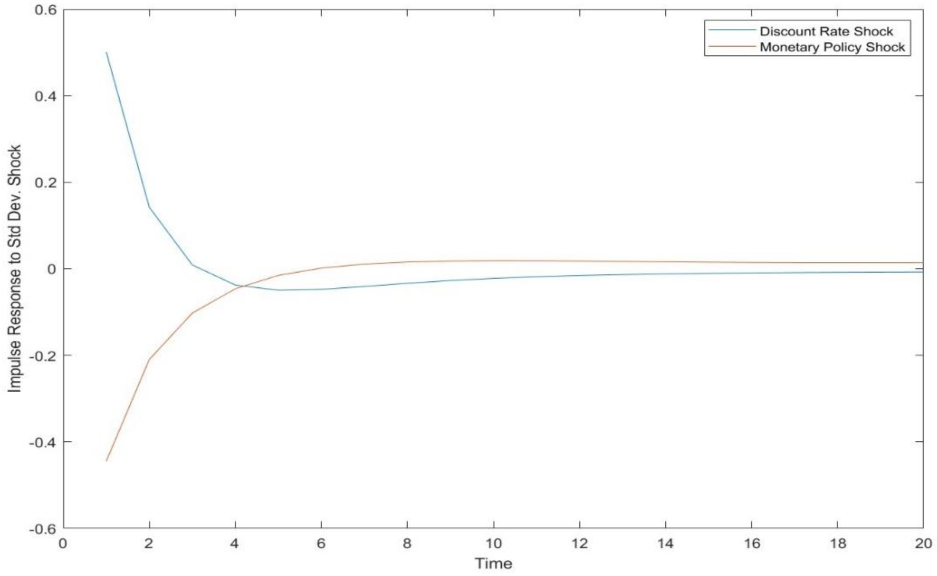


Figure 3 plots the impulse response of consumption to a monetary policy and discount rate shock. A contractionary monetary policy shock leads to a fall in consumption and a contractionary discount rate shock leads to a decrease in savings and subsequent increase in consumption. This figure is relevant because it shows that these shocks generate roughly the same magnitude of response to consumption. Monetary policy has a slightly smaller standard deviation (0.22 vs. 0.35) but the impulses are roughly the same. Thus, one important takeaway from our estimation is that discount rate shocks or shocks that change the saving/consumption decision have a large impact on consumption over a roughly eight quarter horizon.

Figure 4. Impulse Response of Output to Housing Investment and Demand Shocks

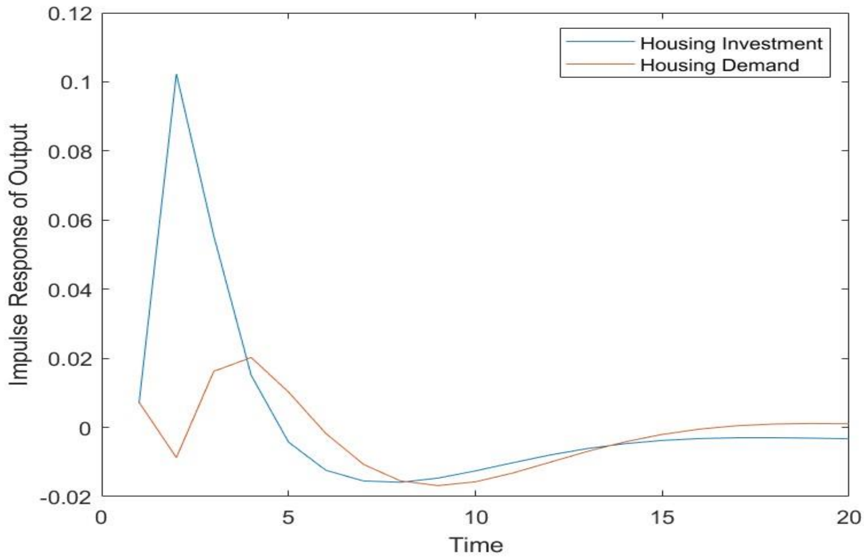


Figure 4 plots the impulse response of output to a housing investment (supply side) and a housing demand shock. Both shocks have relatively large impacts on output. This is due to the estimated elasticity between housing and consumption (1.25). The initial increase to a positive shock is due to the fraction of output attributable to the housing stock. The subsequent decline, while modest, is due to the over-investment in periods 1-5. Households that work in the housing sector take more leisure time after the positive shock, leading to a decline in housing supply. This increases the price of housing, resulting in a decline in housing demand beginning in period 8.

The study also analyses the welfare condition using non-financial variables, such as, productivity, Government spending, investment, the discount factor, Central Bank policy rate, inflation, risk premium, housing supply, and demand. The welfare condition from the non-financial variables were derived using the second order condition. Table 3 shows posterior estimates for both weak and strong financial friction priors. According to the results obtained, a negative shock to productivity will result in a decline in output, a falloff in consumption and rise in inflation. In terms of Government spending, a one standard deviation shock will lead to a decrease in output and consumption, while contributing to higher inflation. Further, a shock the Central Bank Discount rate will yield similar results, led by a reduction in consumption.

Table 6: Welfare Effect of Non-Financial Variables (%)												
Variable Shocks	$\triangle$ Output		$\triangle$ Consumption		$\triangle$ Investment		Inflation		Housing Inflation		$\triangle$ Housing Investment	
	Priors	***Posterior***	***Priors***	***Posterior***	***Priors***	***Posterior***	***Priors***	***Posterior***	***Priors***	***Posterior***	***Priors***	***Posterior***
Productivity	9.5	8.7	4.5	3.9	1.5	1.5	19.8	19.5	0.4	0.3	1.4	1.3
Discount Factor	11.7	11.7	26.6	26.4	0.9	0.9	0.8	0.8	1.1	1.1	1.6	1.7
Govt. Spending	10.9	10.7	1.7	1.7	0.0	0.0	3.0	2.8	0.3	0.3	0.8	0.8
Investment	22.3	22.8	11.7	12.1	65.0	64.4	7.3	7.6	0.2	0.2	1.0	1.1
Policy Rate	21.3	21.0	24.0	23.6	8.2	8.0	1.9	1.9	8.7	8.7	18.6	18.6
Inflation markup	11.4	11.7	12.7	12.9	8.4	8.5	33.3	32.8	0.3	0.4	2.2	2.3
Housing Demand	0.2	0.2	4.5	4.6	0.2	0.2	4.7	4.5	25.6	25.3	64.4	64.1
Housing Supply	1.2	1.1	2.0	2.0	0.1	0.1	0.7	0.7	62.3	62.6	8.2	8.2
Risk Premium	6.9	7.6	5.0	5.7	13.6	14.3	8.5	9.7	0.1	0.2	0.4	0.5
Other Financial	2.6	2.5	5.0	4.8	0.5	0.5	0.4	0.3	0.9	0.9	0.8	0.8
Source: Authors' Estimates												

Section 6. Policy Implications and Conclusion

6.1 Policy Implications

Indications are that this paper is the first to estimate a DSGE model with Bahamian data. The model introduces many non-standard, financial frictions that the data suggest are important for understanding consumption, investment and output in The Bahamas. The weak pass-through effects of monetary policy are observed in the results. For example, the impact of a monetary policy shock is on-par with a shock to the discount factor. This would not be the case if monetary policy had substantial pass-through effects.

Output has a significant response to the housing market and related shocks, as shown by the variance decomposition results and impulse response functions. Macroprudential regulation, as opposed to more standard policy action, has an outsized impact on Bahamian macro aggregates.

Importantly, Table 6 further corroborates the main thesis of the paper, that non-standard shocks are most important in understanding welfare, with the discount factor explaining the largest change in welfare. These shocks are typically excluded in standard New Keynesian models. The results suggest that the financial sector is an integral part of the Bahamian economy and should be modelled.

6.2 Conclusion

The key findings of this study indicate that macroprudential indicators and macroprudential shocks are important considerations for understanding the dynamics of the Bahamian economy, and drivers of welfare ex-ante and ex-post recessionary periods. This conclusion is underpinned by the decomposition of consumption, which reveals that discount shocks are the main driver for variation in consumption, especially during periods of recession or negative consumption growth; while standard shocks drive the historical decomposition of output. Also of note, is the large impact on consumption from discount rate shocks and those that impact the saving/consumption decision.

Moreover, the comparison of a model with macroprudential frictions to one without, revealed that the latter contributes to “policy mistakes” thereby reinforcing the importance of macroprudential indicators for macroeconomic analysis and policymaking on the part of the monetary authority.

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Appendix A: Derivation of Equilibrium Equations

Detrending

For detrending purposes, we define new variables such as: $\xi_t = \Xi_t/\gamma^{-\sigma_c t}, h_t = H_t/\gamma^t, k_t = K_t/\gamma^t, k_t^h = K_t^h/\gamma^t, c_t = C_t/\gamma^t, w_t = W_t/(P_t\gamma^t), \bar{\beta} = \beta \cdot \gamma^{-\sigma_c}, \bar{\beta}' = \beta' \cdot \gamma^{-\sigma_c}$, where Ξ_t is the Lagrange multiplier with regard to the budget constraint. Then the first order conditions of patient households are

$$\xi_t =$$

$$\left[(1-\psi)\left(c_t - \frac{\lambda}{\gamma}c_{t-1}\right)^{1-\sigma_h} + \psi\epsilon_t^{\psi}h_t^{1-\sigma_h}\right]^{\frac{\sigma_h-\sigma_c}{1-\sigma_h}}(1-\psi)\left(c_t - \frac{\lambda}{\gamma}c_{t-1}\right)^{-\sigma_h}\exp\left[\frac{\sigma_c-1}{1+\sigma_l}L_t^{1+\sigma_l}\right],$$

$$w_t^h = -\frac{U_{l,t}}{\Xi_t\gamma^t} = \left[(1-\psi)\left(c_t - \frac{\lambda}{\gamma}c_{t-1}\right)^{1-\sigma_h} + \psi\varepsilon_t^{\psi}h_t^{1-\sigma_h}\right]\frac{1}{1-\psi}\left(c_t - \frac{\lambda}{\gamma}c_{t-1}\right)^{\sigma_h}L_t^{\sigma_l},$$

$$q_t^h = E_t\bar{\beta}\frac{\varepsilon_{t+1}^{\beta}}{\varepsilon_t^{\beta}}\cdot\frac{\xi_{t+1}}{\xi_t}q_{t+1}^h(1-\delta_h) + \frac{\psi\varepsilon_t^{\psi}}{1-\psi}\cdot\frac{(c_t - \lambda/\gamma c_{t-1})^{\sigma_h}}{(h_t)^{\sigma_h}}$$

The first order conditions of impatient household are

$$1 = E_t\bar{\beta}'\frac{\varepsilon_{t+1}^{\beta}}{\varepsilon_t^{\beta}}\frac{\xi'_{t+1}}{\xi'_t}\frac{R_t^b}{\Pi_{t+1}} + \Omega_t,$$

$$q_t^h = E_t\bar{\beta}'\frac{\varepsilon_{t+1}^{\beta}}{\varepsilon_t^{\beta}}\cdot\frac{\xi'_{t+1}}{\xi'_t}q_{t+1}^h(1-\delta_h) + \frac{\psi\epsilon_t^{\psi}}{1-\psi}\cdot\frac{(c'_t - \lambda/\gamma c'_{t-1})^{\sigma_h}}{(h'_t)^{\sigma_h}} + \Omega_t m E_t q_{t+1}^h.$$

where we define Λ_t as the Lagrange multiplier with regard to debt constraint and Ω_t as the ratio of Lagrange multipliers, $\Omega_t \equiv \Lambda_t/\Xi'_t$.

In housing goods producer's problem, the law of motion for housing can be written as

$$h_t^a - (1-\delta_h)h_{t-1}^a/\gamma = ih_t = A_t^h ik_t^h,\ (h_t^a = h_t + h'_t)$$

and the optimality condition is

$$Q_t^h A_t^h - \left\{1 + s^h\left(\frac{ik_t^h/\gamma}{ik_{t-1}^h}\right)\right\} - s^{h'}\left(\frac{ik_t^h/\gamma}{ik_{t-1}^h}\right)\frac{ik_t^h/\gamma}{ik_{t-1}^h} = 0$$

Steady State

The following describes the steady-state of the economy with respect to the variables in the housing market. Since housing goods can be transformed from consumption goods with no cost, the steady state price of housing goods in terms of consumption goods is 1. From the first-order condition, we obtain

$$\{1 - \bar{\beta}(1-\delta_h)\} = \frac{\psi}{1-\psi}\cdot\frac{(1-\lambda/\gamma)^{\sigma_h}c^{\sigma_h}}{h^{\sigma_h}}$$

$$c=\left[\frac{1-\psi}{\psi}\cdot\frac{1-\bar{\beta}(1-\delta_h)}{(1-\lambda/\gamma)^{\sigma_h}}\right]^{\frac{1}{\sigma_h}}h=\mu_{ch}\cdot h.$$

and

$$\left\{1-\bar{\beta}'(1-\delta_h)-\left(1-\frac{\bar{\beta}'}{\bar{\beta}}\right)m\right\}=\frac{\psi}{1-\psi}\cdot\frac{(1-\lambda/\gamma)^{\sigma_h}(c')^{\sigma_h}}{(h')^{\sigma_h}},$$

$$c'=\left[\frac{1-\psi}{\psi}\cdot\frac{1-\bar{\beta}'(1-\delta_h)-\left(1-\frac{\bar{\beta}'}{\bar{\beta}}\right)m}{(1-\lambda/\gamma)^{\sigma_h}}\right]^{\frac{1}{\sigma_h}}h'=\mu'_{ch}\cdot h',$$

Define $\Upsilon\equiv 1-\frac{i}{y}-g$, we obtain

$$\Upsilon=\frac{c}{y}+\frac{c'}{y}+\frac{ik^h}{y}$$

For the housing production side,

$$ik^h=(1-(1-\delta_h)/\gamma)h^a=\mu_{ik}h^a$$

Also, from impatient households' budget constraint,

$$\frac{c'}{y}+\frac{(1-\mu)g}{y}+\frac{1-(1-\delta_h)/\gamma}{\mu'_{ch}}\cdot\frac{c'}{y}-\left(1-\frac{1}{\bar{\beta}\gamma}\right)\frac{m}{\mu'_{ch}}\frac{c'}{y}=(1-\alpha)(1-\mu)$$

$$\Rightarrow\frac{c'}{y}=\left[(1-\alpha)(1-\mu)-\frac{(1-\mu)g}{y}\right]\Big/\left[1+\frac{1-(1-\delta_h)/\gamma}{\mu'_{ch}}-\left(1-\frac{1}{\bar{\beta}\gamma}\right)\frac{m}{\mu'_{ch}}\right]$$

$$\frac{c}{y}=\left[\Upsilon-\left(1+\frac{\mu_{ik}}{\mu'_{ch}}\right)\frac{c'}{y}\right]\Big/\left(1+\frac{\mu_{ik}}{\mu_{ch}}\right)$$

Log-linearization Around the Steady State

In the following text, log-linear variables are denoted by hat. Marginal utility of consumption is

$$\hat{\xi}_t=\frac{\sigma_h-\sigma_c}{1-\sigma_h}\hat{j}^t+(\sigma_c-1)L^{1+\sigma_l}\hat{L}_t-\sigma_h\frac{\hat{c}_t-\lambda/\gamma\hat{c}_{t-1}}{1-\lambda/\gamma}$$

where $\hat{j}_t$ is defined by

$$\hat{j}_t=\frac{(1-\tau)(1-\sigma_h)}{1-\lambda/\gamma}(\hat{c}_t-\lambda/\gamma\hat{c}_{t-1})+\tau(1-\sigma_h)\hat{h}^t+\tau\hat{\epsilon}_t^{\psi},\ \tau=\frac{\psi}{(1-\psi)(1-\lambda/\gamma)\left(\frac{c}{h}\right)^{1-\sigma_h}+\psi}$$

Note $L^{1+\sigma_l}$ can be written as

$$L^{1+\sigma_l} = \frac{w^h L}{c}\left(1-\frac{\lambda}{\gamma}\right)^{-\sigma_h} / \left[\left(1-\frac{\lambda}{\gamma}\right)^{1-\sigma_h} + \frac{\psi}{1-\psi}\left(\frac{h}{c}\right)^{1-\sigma_h}\right]$$

The first-order condition becomes

$$\begin{aligned} \hat{q}_t^h = \quad & \bar{\beta}(1-\delta_h)E_t\left[\hat{q}_{t+1}^h - \hat{R}_t + \hat{\pi}_{t+1}\right] \\ & + \frac{1-\bar{\beta}(1-\delta_h)}{1-\lambda/\gamma}\cdot\sigma_h\left[\hat{c}_t - \frac{\lambda}{\gamma}\hat{c}_{t-1} + \left(1-\frac{\lambda}{\gamma}\right)\left(\frac{1}{\sigma_h}\hat{\varepsilon}_t^{\psi} - \hat{h}_t\right)\right]. \end{aligned}$$

and

$$\begin{aligned} \hat{\Omega}_t &= \frac{-\bar{\beta}'/\bar{\beta}}{1-\bar{\beta}'/\bar{\beta}}E_t\left[\hat{\xi}'_{t+1} - \hat{\xi}'_t + \hat{R}_t^b + \hat{\varepsilon}_{t+1}^{\beta} - \hat{\varepsilon}_t^{\beta} - \hat{\pi}_{t+1}\right] \\ \Rightarrow E_t\left[\hat{\varepsilon}_{t+1}^{\beta} - \hat{\varepsilon}_t^{\beta} + \hat{\xi}'_{t+1} - \hat{\xi}'_t\right] &= \frac{1-\bar{\beta}'/\bar{\beta}}{-\bar{\beta}'/\bar{\beta}}\hat{\Omega}_t - E_t\left[\hat{R}_t^b - \hat{\pi}_{t+1}\right] \\ \hat{q}_t^h = E_t\bar{\beta}'(1-\delta_h)&\left[\hat{\varepsilon}_{t+1}^{\beta} - \hat{\varepsilon}_t^{\beta} + \hat{\xi}'_{t+1} - \hat{\xi}'_t + \hat{q}_{t+1}^h\right] + m\left(1-\bar{\beta}'/\bar{\beta}\right)E_t\left[\hat{\Omega}_t + \hat{q}_{t+1}^h\right] \\ &+\left[1-\bar{\beta}'(1-\delta_h) - m\left(1-\bar{\beta}'/\bar{\beta}\right)\right]\cdot\sigma_h\left[\frac{1}{\sigma_h}\hat{\epsilon}_t^{\psi} - \hat{h}'_t + \frac{\hat{c}'_t - \frac{\lambda}{\gamma}\hat{c}'_{t-1}}{1-\frac{\lambda}{\gamma}}\right] \end{aligned}$$

The budget constraint of the borrowing household becomes

$$\begin{aligned} & \frac{c'}{y}\hat{c}'_t + (1-\mu)\hat{g}_t + \frac{c'/\mu'_{ch}}{y}\left(\hat{q}_t^h + \hat{h}_t\right) + \frac{m}{\gamma\bar{\beta}}\frac{c'/\mu'_{ch}}{y}\left(\hat{b}_{t-1} + \hat{R}_{t-1}^b - \hat{\pi}_t\right) \\ = \quad & (1-\alpha)(1-\mu)\hat{y}_t + \frac{c'/\mu'_{ch}}{y}\frac{1-\delta_h}{\gamma}\left(\hat{q}_t^h + \hat{h}'_{t-1}\right) + \frac{mc'/\mu'_{ch}}{y}\left(\hat{b}_t\right), \end{aligned}$$

and LTV constraint becomes

$$\hat{b}_t = \hat{\varepsilon}_t^{dbt} + E_t\hat{q}_{t+1}^h + \hat{h}'_t.$$

Law of motion for the gross housing goods and the optimality condition for the housing goods producing firms are given by

$$\begin{gathered} \left[1/\left(1-\frac{1-\delta_h}{\gamma}\right)\right]\hat{h}_t^a - \left[\frac{1-\delta_h}{\gamma}/\left(1-\frac{1-\delta_h}{\gamma}\right)\right]\hat{h}_{t-1}^a = \hat{a}_t^h + i\hat{k}_t^h \\ \hat{q}_t^h + \hat{a}_t^h - S''(\gamma)^2\left(i\hat{k}_t^h - i\hat{k}_{t-1}^h\right) = 0. \end{gathered}$$

Aggregate resource constraint is given by

$$\hat{y}_t = \frac{c}{y}\hat{c}_t + \frac{i}{y}\hat{\imath}_t + \frac{ik}{y}\hat{\imath}_t + r^{kss}\frac{k}{y}\hat{z}_t + \hat{\varepsilon}_t^g$$

Regarding the financial frictions in the business sector, the marginal productivity of capital $\hat{x}_t$ is given by

$$\hat{x}_t = \hat{l}_t + \hat{w}_t^a - \hat{k}_t$$

where $\hat{w}_t^a$ is the weighted average real wage of patient and impatient household. Then the return on capital is defined by

$$\hat{r}_t^k = \left(1 - \frac{1-\delta}{R^k}\right)\hat{x}_t + \frac{1-\delta}{R^k}\hat{q}_t - \hat{q}_{t-1}$$

where $\hat{q}_t$ is the price of capital. Given this definition of return on capital, the log-linear form of financial accelerator equation is

$$E_t\hat{r}_{t+1}^k - \hat{r}_t^f = \chi^e\left(\hat{q}_t + \hat{k}_t^p - n\hat{w}_t\right) + \hat{\varepsilon}_t^{rp}$$

where χ^e is the parameter that represents the elasticity of the external finance premium with regards to the entrepreneur's net worth. The law of motion for the entrepreneur's net worth is

$$\gamma n\hat{w}_t = \vartheta\left[\frac{K^p}{NW}\left(R^k\hat{r}_t^k - R^f\hat{r}_{t-1}^f\right) + \left(R^K - R^f\right)\frac{K^p}{NW}\left(\hat{q}_{t-1} + \hat{k}_{t-1}^p\right) + R^f n\hat{w}_{t-1}\Big)\right] + \hat{\varepsilon}_t^{nw}$$

Regarding the financial friction for the financial intermediary sector, we have the relationship between the bank spread and the distance-to-default,

$$\widehat{bs}_t = \chi^f \cdot \widehat{DD}_t + \hat{\varepsilon}_t^{bs}$$

and the relationship between the bank distance-to-default and the expected housing price and capital price,

$$\widehat{DD}_t = \rho^{dd}\widehat{DD}_{t-1} + \chi^{dd,H}E_t\hat{q}_{t+1}^h + \chi^{dd,Q}\hat{q}_t + \hat{\varepsilon}_t^{dd}$$

Other equilibrium conditions

Non-financial friction part of the SW-FF model is similar to the SW model. The production function of the economy is given by

$$\hat{y}_t = \phi_p\left(\alpha\hat{k}_t + (1-\alpha)\hat{l}_t + \hat{\varepsilon}_t^a\right)$$

and non-residential capital service is defined by

$$\hat{k}_t = \hat{k}_{t-1}^p + \hat{u}_t$$

where $\hat{k}_t^p$ is physical capital stock and $\hat{u}_t$ is utilization rate. The following relationship exists between the marginal cost of production and the wage and marginal productivity of capital,

$$\widehat{mc}_t + \hat{\varepsilon}_t^a = \alpha\hat{x}_t + (1-\alpha)\hat{w}_t^a.$$

Law of motion for the physical capital stock is given by

$$\hat{k}_t^p = (1-\delta)/\gamma \hat{k}_{t-1}^p + (1-(1-\delta)/\gamma)\hat{\imath}_t + (1-(1-\delta)/\gamma)\varphi\gamma^2\hat{\varepsilon}_t^i$$

where δ is the depreciation rate. From the optimality condition for the capital utilization, we have the relationship between the the marginal productivity of capital and the level of utilization,

$$\hat{u}_t = \frac{1-\Psi}{\Psi}\hat{x}_t$$

Capital producer's first order condition with regards to investments gives us the following optimality condition

$$\hat{\imath}_t = \frac{1}{1+\beta\gamma^{(1-\sigma_c)}}\hat{\imath}_{t-1} + \frac{\beta\gamma^{(1-\sigma_c)}}{1+\beta\gamma^{(1-\sigma_c)}}E_t\hat{\imath}_{t+1} + \frac{1}{\phi\gamma^2(1+\beta\gamma^{(1-\sigma_c)})}\hat{q}_t + \hat{\varepsilon}_t^i$$

There is a Calvo type of nominal rigidity in intermediate goods production, as only a certain fraction of intermediate good producers can choose the optimal sales price. The price of producers who cannot optimize are partially indexed to the past inflation. Optimization by price-setting producers leads to the following New Keynesian Phillips curve,

$$\begin{aligned}\hat{\pi}_t = \quad & \frac{\beta\gamma^{(1-\sigma_c)}}{1+\iota_p\beta\gamma^{(1-\sigma_c)}}E_t\hat{\pi}_{t+1} + \frac{\iota_p}{1+\iota_p\beta\gamma^{(1-\sigma_c)}}\hat{\pi}_{t-1} \\ & - \frac{\left(1-\beta\gamma^{(1-\sigma_c)}\xi_p\right)\left(1-\xi_p\right)}{\left(1+\iota_p\beta\gamma^{(1-\sigma_c)}\right)\left(1+\left(\phi_p-1\right)\epsilon_p\right)\xi_p}\hat{\mu}_t^p + \hat{\varepsilon}_t^p\end{aligned}$$

where $\iota_p, \xi_p, \epsilon_p$ are the degree of indexation to past inflation, the degree of price stickiness, and the curvature of Kimball goods market aggregator. Also, the markup in the intermediate goods production $\hat{\mu}_t^p$ equals

$$\hat{\mu}_t^p = \alpha\left(\hat{k}_t - \hat{l}_t\right) - \hat{w}_t + \hat{\varepsilon}_t^a$$

There is also nominal rigidity in wage decision, as only a fraction of labor unions can optimally reset nominal wage, and the other fraction only partially index their wage to the past wage. Optimality conditions lead to the expression for real wage for patient and impatient households,

$$\hat{w}_t = \frac{\beta\gamma^{(1-\sigma_c)}}{1+\beta\gamma^{(1-\sigma_c)}}(E_t\hat{w}_{t+1} + E_t\hat{\pi}_{t+1}) + \frac{1}{1+\beta\gamma^{(1-\sigma_c)}}(\hat{w}_{t-1} + \iota_w\hat{\pi}_{t-1})$$
$$-\frac{1+\beta\gamma^{(1-\sigma_c)}\iota_w}{1+\beta\gamma^{(1-\sigma_c)}}\hat{\pi}_t - \frac{\left(1-\beta\gamma^{(1-\sigma_c)}\xi_w\right)(1-\xi_w)}{(1+\beta\gamma^{(1-\sigma_c)})(1+(\varphi_w-1)\epsilon_w)\xi_w}\hat{\mu}_t^w + \hat{\varepsilon}_t^w$$
$$\hat{w}_t' = \frac{\beta\gamma^{(1-\sigma_c)}}{1+\beta\gamma^{(1-\sigma_c)}}(E_t\hat{w}_{t+1}' + E_t\hat{\pi}_{t+1}) + \frac{1}{1+\beta\gamma^{(1-\sigma_c)}}(\hat{w}_{t-1}' + \iota_w\hat{\pi}_{t-1})$$
$$-\frac{1+\beta\gamma^{(1-\sigma_c)}\iota_w}{1+\beta\gamma^{(1-\sigma_c)}}\hat{\pi}_t - \frac{\left(1-\beta\gamma^{(1-\sigma_c)}\xi_w\right)(1-\xi_w)}{(1+\beta\gamma^{(1-\sigma_c)})(1+(\varphi_w-1)\epsilon_w)\xi_w}\hat{\mu}_t^{w'} + \hat{\varepsilon}_t^w$$

where $\iota_w, \xi_w, \epsilon_w$ are the degree of indexation to past wage, the degree of wage stickiness,and the curvature of Kimball labor market aggregator. Also, the mark ups in the wage contract $\hat{\mu}_t^w, \hat{\mu}_t^{w'}$ equal

$$\hat{\mu}_t^w = \hat{w}_t - \sigma_l\hat{l}_t - \frac{1}{1-\lambda/\gamma}(\hat{c}_t - \lambda/\nu\hat{c}_{t-1}),\ \hat{\mu}_t^{w'} = \hat{w}_t' - \sigma_l\hat{l}_t' - \frac{1}{1-\lambda/\gamma}(\hat{c}_t' - \lambda/\nu\hat{c}_{t-1}')$$

Monetary policy sets the nominal interest rate $\hat{r}_t^N$ in a way that reacts to inflation, output gap and changes in output gap. Output gap is defined by the difference between the current output $(\hat{y}_t)$ and the flexible-price, flexible-wage economy output $(\hat{y}_t^*)$.

$$\hat{r}_t^N = \rho\hat{r}_{t-1}^N + (1-\rho)\left[r_\pi\hat{\pi}_t + r_y(\hat{y}_t - \hat{y}_t^*)\right] + r_{\Delta y}[(\hat{y}_t - \hat{y}_t^*) - (\hat{y}_{t-1} - \hat{y}_{t-1}^*)] + \hat{\varepsilon}_t^r$$

Regarding exogenous processes, productivity shock $\hat{\varepsilon}_t^a$, discount factor shock $\hat{\varepsilon}_t^\beta$, investment specific shock $\hat{\varepsilon}_t^i$, monetary policy shock $\hat{\varepsilon}_t^r$, lending stand shock $\hat{\varepsilon}_t^{dbt}$, firm net worth shock $\hat{\varepsilon}_t^{nw}$, risk premium shock $\hat{\varepsilon}_t^{rp}$, bank spread shock $\hat{\varepsilon}_t^{bs}$, distance-to-default shock $\hat{\varepsilon}_t^{dd}$, housing demand shock $\hat{\varepsilon}_t^\psi$, housing supply shock $\hat{a}_t^h$ follow AR(1) process. Government spending shock $\hat{\varepsilon}_t^g$ follows AR(1) process with a correlation with productivity shock. Inflation markup shock $\hat{\varepsilon}_t^p$ and wage markup shock $\hat{\varepsilon}_t^w$ follow $\mathrm{ARMA}(1,1)$ process.

Appendix B: Data

Definition of Bahamian Data (Source: Central Bank of The Bahamas)

Consumption = LN[(PCEC/GDPDEF)] × 100

Residential investment = LN[(FPIR/GDPDEF)/LNSindex] × 100

Output = LN(GDPC96/LNSindex) × 100

Hours = LN[(PRS85006023 × CE16OV/100)/LNSindex] × 100

Inflation = LN(GDPDEF/GDPDEF(-1)) × 100

Real wage = LN(PRS85006103/GDPDEF) × 100

Interest rate = 10-Year BGS rate /4

Definition of U.S. Data

Firm leverage = LN[(Firm Asset)/(Firm Asset-Firm Debt)], demeaned

Distance to default = LN(Z-score Distance to Default)

Interest rate spread = (Federal Funds Rate - 1m Euro-Dollar Deposit Rate)/4

Risk spread = (Moody's BAA-10 Year Treasury Spread)/4 - Interest rate spread

Housing price = LN[(Housing Price Index/GDPDEF)/(Housing Price Index(-1)/GDPDEF(-1)] × 100