

The real and the residual demands for money: a
reconstruction of the theory of money-demand

by

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Caribbean economists, like their counterparts elsewhere, are divided on the question of the empirical existence of the money demand function. On the one hand studies conducted by Bourne⁽¹⁾, McIntyre⁽²⁾ and Ramsaran⁽³⁾ while confirming the importance of income have not succeeded in establishing the causative influence of other variables (interest rates, prices, rates of return on complementary assets etc.) nor have those studies convincingly established either (a) stability of the overall relationship or (b) unidirectional causality in any statistical sense⁴. On the other hand, studies of the demand for money on Barbadian data for the period 1966-'79 and on data for Trinidad and Tobago over a similar period appear to support the hypothesis of a stable income velocity in the latter country and the existence of a well-defined function in both countries⁵. These latter studies like the former set make no vigorous econometric effort to identify the direction of causation in the time series analysed. More important, none of the authors has sought to question or to discuss the theoretical foundations of the demand function for money. It appears to have been assumed that those theoretical foundations (Keynesian or neo-classical) were unimpeachable

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and that the ability to forecast and to control monetary aggregates in Caribbean-type economies hinged on the empirical discovery of the unmodified function⁶.

"...if the universality of the money-demand function is to be accepted, its existence must be supported by evidence from countries large and small, rich and poor, industrialised and unindustrialised, open and not so open" (McLean, op.cit., p. 156).

"If the income velocity is a parameter of the system and the demand for money as an asset is a stable function of wealth/income and the opportunity cost of holding cash, then variations in the supply of money will be felt in the goods market thus creating excess demand in times of monetary expansion".
(St. Cyr, op.cit., p. 620)

"We propose a functional form (of money demand) similar to those in wide use in the developed countries, but make adjustments for institutional factors and data limitations" (McIntyre, op.cit., p. 188).

The ambiguous results that have to date been produced by this mode of investigating Caribbean economies raise a number of questions:-

- (1) have the different results been due to differences or to changes in institutional/environmental factors not fully allowed for in the investigations?
- (2) have the different results been due to data limitations?
- (3) have they been due to differences in testing methods or in econometric techniques?
- (4) could the results have been more instructive if tests had been conducted on rival or alternate theories of the demand for money?

Question (4) appears to us to be fundamental and in the remainder of this paper we shall be concerned with examining a range of possible responses to it.

In inaugurating the modern theory of the demand for money it will be recalled that Keynes began by turning on its head the velocity concept of the traditional quantity theory and splitting the ratio so derived into two, namely, k_1 , the average proportion of cash deposits held against annual aggregate money income and k_2 , the average proportion of bank-money held against business deposits⁷. Initially, Keynes' focus appeared to have been centred on the empirical behaviour of these ratios in the advanced capitalist economies of the United States and the United Kingdom. There was no doubt in his mind that both k_1 and k_2 "...are held for the purpose of making payments - in distinction from Savings deposits which are held for a different purpose."⁸ However, lacking the detailed national income, industrial and financial data required for identifying the payments corresponding to k_1 and to k_2 , Keynes did not think he could go beyond a mere outline of the likely behaviour of these ratios. On the assumption that annual income deposits would be identical with annual national income, k_1 was expected to be stable, in-so-far as it is determined by stable habits and customs relating especially to the time patterns of receipts and disbursements of income, while k_2 was expected to be volatile, reflecting the instability of speculative and financial transactions. On the other hand, business transactions arising out of 'productive functions', viz. factor income payments and intermediate transactions, were likely to impose a stable influence on k_2 . This

stabilizing influence, Keynes was convinced, would be completely overwhelmed by the volatile elements thus rendering k_2 in general a highly unstable parameter.

Building on the foundations of his "Treatise...", Keynes in "The General Theory..." constructs an aggregate demand for money function on the idea of liquidity preference. The demands for money held separately in the form of income, business and savings deposits are pooled into a single aggregate M which is now expressed as a function of aggregate income, Y , and the average rate of interest r , with $\partial M / \partial Y > 0$, $\partial M / \partial r < 0$. The assumption of constancy of the income velocity of money which loomed large in the classical quantity theory is now relegated to a special case in which the impact of the rate of interest on the demand for money can be ignored - an event likely to occur in "very abnormal circumstances" only^a.

More important for our present discussion is the fact that Keynes in "The General Theory..." constructs the aggregate demand for money function on the foundation of subjective value theory (however primitive) rather than by associating individual holdings of money with its use as means of payment, which use, as we saw him admitting in the "Treatise...", constitutes the purpose for holding money.

The neo-classical assault on Keynesian monetary theory led by Milton Friedman deepened and extended the roots of the theory of money - demand in utility theory and the theory of choice. In doing so, the linkages between money

and actual business or income transactions are all but lost. The theory of money demand is now located in the general theory of capital since this demand is seen as representing, on the one hand, the demand for an asset by individual owners of wealth and, on the other hand, a source of productive services to businessmen who combine money with other inputs to produce goods and services as commodities. Given utility maximisation by the individual wealth holder the amount of money held by him will be such that the rate at which he can substitute it for other forms of wealth is equal to the rate at which he is willing to do so. The yield per unit of money itself accrues subjectively to the owner in the form of convenience, 'security', etc. while the magnitude of this yield in real terms "depends on the volume of goods that unit corresponds to, or on the general price level which we may designate by P ".¹⁰ Thus income and the rate of interest in the neo-classical money demand function do not convey, respectively, the transactions/precautionary and speculative motives for holding cash as they did in Keynes' formulation; on the contrary they are to be interpreted together as indicators of the magnitude of an individual's wealth. Fully elaborated, the neo-classical demand for money function relates the demand for money to its own yield (the average price level), the real yields on competing or complementary assets (e.g. bonds, equities, physical goods), the ratio of non-human to human wealth (as a crude indicator of the rate-of substitution between these

two forms of wealth) and total wealth (as measured by the stock flow indicator Y/r or by permanent income). If this demand function can, like other demand functions, be considered homogenous of degree zero in prices and income, the neo-classical quantity theory of money predicts:

$$Y = v (. . .) M$$

where v , the income velocity of circulation of money, is no longer a mechanical constant, as in the traditional quantity theory, but a parameter dependent on a vector of real yields on financial and non-financial assets summarised in (. . .). As is standard in subjective value theory no justification for aggregating across individual preference fields is deemed necessary. The proof of the existence of the aggregate demand function is left to depend upon the results of empirical tests. Differences between "quantity theorists" and the rest, from here on, resolve themselves into differences in the a priori specifications of the parameters of the function and on its identifiability in general.

A gamut of studies of the demand function for money have followed in the wake of Friedman's "restatement". Up to the late '60's these studies seemed capable of generating the following conclusion: "As far as the theory of the demand for money is concerned, it has been concluded that the traditional method of demand theory, whereby a utility function is postulated but not examined in any detail seems to work as well for money as it does for other goods.

Detailed analysis of transactions and speculative motives so far appear to have added little if anything to the stock of empirically valid predictions about the demand for money."¹¹ However, by the end of the '70's continued research had brought to light shifts in the observed relationship so significant as to call into question its very basis in theory. "When important issues like the stability of the demand for money function begins to depend, for example, on just which interest rate or rates one uses to proxy the opportunity cost of holding money, I believe that the correct conclusion is not that the variable which provides the best fit this time around is the right one, but that our knowledge of the details of the relationship is more fragile than we thought".¹² This uncertainty applied especially to tests of the relationship conducted on time series.

Meanwhile, theoretical studies of the transactions and precautionary demands for cash employing utility optimising models of behaviour had suggested that these demands were much more interest-sensitive than Keynes had originally hypothesized and that under moderately restrictive assumptions the optimal magnitudes of both demands would tend to vary less than proportionately with variations in income.¹³ However, in cross-sectional studies of the demand for money conducted by Meltzer on U.S. data for individual firms during selected years of the period 1938-1957 neither the hypothesis of economies nor of diseconomies of scale in the holding of cash balances across firms could be discerned

although the interest rate was found to be an important determining factor in firms' cash demands and economies/diseconomies of scale did show up in some industries.¹⁴ It is important to note here that Meltzer conducted his tests on total cash balances held by firms (viz. transactions plus liquidity balances) and that, unsurprisingly, the demand elasticity with respect to sales revenues (the proxy for wealth in the cross-sectional testing of the money-demand function) was found to depend on distributional effects across industries as well as other factors. Meltzer's conclusion is worth quoting: "Changes in the parameters of the cross-section demand functions are consistent with the view that the yields on alternative assets influence the composition of business balance sheets. But my hypothesis suggests that interest rates are not sufficient to explain all of the short term fluctuations in velocity within the business sector. Short-run changes in the mix of aggregate output and in the internal yield on a firm's assets appear to be of some consequence".¹⁵ This lead, however, appears not to have been explored in subsequent research.¹⁶ The utility based money-demand function continued to hold sway with the ambiguous empirical results previously described.

In recent years the econometric and theoretic failures of the money-demand function and other key postulates of macroeconomics have led to the view that "...modern

macroeconomic models are of no value in guiding policy and that this condition will not be remedied by modifications along any line which is currently being pursued."¹⁷ Hence the current search for a new macroeconomics and an appropriate macroeconometric framework to accompany it.¹⁸

For the developing countries McKinnon has offered a modification of the neo-classical money-demand function that promises little relief from its current dilemmas. In place of the "competing-asset" effect he proposes that the money-demand function in developing countries is better understood (owing to the fragmented economic structure) as embodying complementary linkages between money and physical capital. If this complementarity can be proxied by the average return on capital, $\bar{\pi}$, we ought to follow through by replacing the rate of return on physical goods in the neo-classical demand function by $\bar{\pi}$ and interpret $\partial m / \partial \bar{\pi} > 0$.¹⁹ No other major change to the function is proposed.

While McKinnon seems to be able to draw major policy conclusions from this modification it is doubtful that his proposed change to the money-demand function has removed the need for fundamental re-examination of the theory of demand for money. Certainly in the Caribbean no economist has as yet tested McKinnon's version of the money-demand function. But in any event it is clear that in so far as such tests would seek to identify an aggregate money-demand function by conventional econometric analysis of time series data they offer no hope of relief from the present difficulties of

macroeconomic theory and macroeconometrics. McKinnon's "modification" of the neo-classical money-demand function is moreover already included in Friedman's "restatement" which makes allowance for the inclusion of yields on substitute and complementary assets within the argument of the function.

Thus the case for fundamental overhaul of the theoretical foundations of the money-demand function remains. For us in the Caribbean this overhaul could be considered as a matter of urgency. Several of our "more developed" economies are currently in a state of economic crisis. This crisis manifests itself in financial circulation and in the sphere of real production. Now more than ever it seems vital to comprehend the nature of the linkages between money and production so as to improve policy formulation especially in matters pertaining to economic growth and the reduction of debt. The theory of the aggregate demand for money has attracted the attention of Caribbean economists not least of all because it appeared to explain those linkages in a rigorously analytical way. This appearance has proved deceptive. True, all empirical tests conducted on Caribbean time series data seem to confirm the importance of income. But since the income effect is open to different interpretations in the Keynesian and neo-classical formulations of money-demand theory the importance of that empirical result remains to be clarified. The empirical findings on the effects of interest rates are

ambiguous and there is no agreement on the stability of the function through time as the time-series data sets employed by investigators are typically scanty, even if they generate robust parameter estimates. Very few cross-sectional studies of the behaviour of Caribbean non-financial enterprises have been conducted but flow-of-funds research has been initiated.²⁰

It is our argument in this paper that the aggregate demand for money function is best comprehended and interpreted when it is constructed on the foundations of the micro-economic theory of the demand for money at the level of the firm or establishment. This presupposes that the demand for money is a demand for means of circulation for purposes of executing financial and non-financial transactions. Keynes in the "Treatise..." had subdivided these transactions as follows:²¹

i) transactions arising out of the division of productive functions including:-

(a) factor income payments; and

(b) intermediate purchases

ii) speculative transactions

iii) financial transactions

We shall call the demand for money associated with (i) the real demand for money, and that associated with (ii), (iii) and expenditure out of income, the residual demand for money. It will be noticed that 1(a) is the exact counterpart within firms' income statements of Income

Deposits as Keynes himself recognised. We shall argue in what follows that the isolation of this double appearance of factor income payments is important in clarifying the linkage between money and production on the one hand and that between the circuit of money and the money market on the other.

While Keynes had linked the transactions and precautionary demands for cash with the level of income or value-added it was never fully explained how and in what ways this linkage was effected. There is only passing reference to "the ancient distinction between the use of money for the transaction of current business and its use as a store of value".²² As regards the amount of money held for speculative purposes we are assured that this "mainly depends on the relation between the current rate of interest and the state of expectation" but that "the rate of interest is a highly psychological phenomenon".²³

In the neo-classical formulation of the theory of the demand for money matters are, as we have seen, no clearer. The demand for money is here treated in a manner similar to the demand for durable goods in the utility-based theory of consumers' demand. The circuit of money has been separated out of the detailed processes of production, exchange and consumption of goods and services in the economy.

This is not so in Marx' analysis of the circuit of money and commodity production. Marx distinguishes between three circuits of money and of commodities in the production

and circulation of commodities in a modern capitalist economy: (1) the circuit of money as money capital

(2) the circuit of money in the circuit of productive capital; and

(3) the circuit of money in the circuit of produced commodities (or of commodity capital).²⁴

These circuits of money describe its use as means of circulation (means of purchase and means of payment) in the production, exchange and distribution of goods and services. The value of the total output of goods and services in any given time period (say one year) exceeds the value of newly produced goods and services by virtue of the reappearance of pre-existing capital values (or means of production) in current aggregate output.²⁵ It follows that a narrow transactions - velocity interpretation of the relationship between M and Y along Keynesian lines is seriously defective.²⁶ Industrial capitalists use money in circuits (1) and (2); commercial capitalists (merchants, money-dealers etc) employ money to conduct transactions summarized by circuit (3). In addition, there is the large and extensive range of transactions connected with circuit (4) the circuit of interest-bearing capital, where money appears to circulate itself and is used almost exclusively as means of payment (credit transactions) rather than means of purchase (spot transactions). The transactions associated with circuit (4) are equivalent to Keynes' Financial Circulation. For both Keynes and Marx the volume

of financial circulation "... is not only highly variable but has no close connection with the volume of output whether of capital goods or of consumption goods, for the current output of fixed capital is small compared with the existing stock of wealth, which... we will call the volume of securities (excluding from this liquid claims on cash) and the activity with which these securities are being passed round from hand to hand does not depend on the rate at which they are being added to."²⁷

Similarly, the average value of the instruments traded in the financial circulation bears no close relation to the short-run price of production of newly produced capital. As regards savings deposits in banks, while Marx merges these with banker's capital and treats both as interest-bearing capital, Keynes quite mistakenly sees no connection between those deposits and bank capital.²⁸ Marx' inclusion of all bank deposits within bank capital is clearly the more realistic conception of this component of financial circulation:- "Bank capital consists of (1) cash money, gold or notes (2) securities. The latter can be subdivided into two parts: commercial paper... and public securities... in short interest-bearing paper, which is however significantly different from (commercial paper). Mortgages may also be included here. The capital composed of these tangible component parts can again be divided into the bankers' invested capital and into deposits, which constitute his banking capital, or borrowed capital... It is evident

that... the actual component parts of the banker's capital... remain unaffected whether the various elements represent the banker's own capital or deposits i.e. the capital of other people".²⁹ However, whether institutions or individuals speculate on the stock market the use of money in financial transactions, entailing as it does the extensive use of credit, adds great instability to the money-production linkage even as it economises the use of money as cash or means of purchase.³⁰ It was Keynes' view that in a modern capitalist economy the volume of financial transactions was not only highly variable but "large enough compared with (those arising out of production and consumption) to confuse the statistics."³¹ In Marx, variation in the volume of financial transactions is related to the phases of the business cycle. The importance of interest-bearing capital in general relates to the division of profit between that portion earned by industrial and commercial capitalists (i.e. profit of enterprise) and the share earned by moneyed-capital. Since both shares add up to aggregate surplus value any increase in the one implies a corresponding decrease in the other. But while aggregate surplus value may exceed gross interest payments by no means could the latter exceed the former, although for particular industries or spheres of production this may occur.³² There is thus for Marx no conduit effect of interest-bearing capital on investment and production which is distinguishable from the conduit effect of money in the form

of equity capital. Money is certainly required in the process of surplus-value creation but its role in this process has nothing to do with its quality but rather with its real volume and the fact that workers separated from ownership of the means of production, require regular payments in the form of money in order to reproduce their labour power.

Marx' analysis of money therefore draws attention to its technical usage in the processes of production, distribution and exchange rather than to its use in financial circulation even though, like Keynes, he acknowledges that its use in the latter role could predominate during certain phases of the business cycle.

In the Caribbean, while considerable financial deepening has occurred in recent years, especially among the MDC's, the use of money for financial circulation is decidedly less ramified and extensive than its technical use in the production and circulation of goods and services. For example, while the domestic savings rate in Trinidad and Tobago has exceeded 40% in recent years "monetary assets still predominate in the financial asset portfolios of the private non-financial sector... broad money comprised 52% of total financial assets of Trinidad and Tobago households in 1976, actuarial and trust funds accounting for an additional 32%."33 Between 1980 and 1984 holdings of private local securities varied between 2-3% of total financial assets of all financial institutions, mortgage loans comprised 8-15%,

holdings of local government securities amounted to 2.4 - 3.2% while holdings of foreign assets (mainly government securities) although falling between those years varied between 19.0% - 49.2%. Broad money (currency plus demand, savings and time deposits) is estimated at between 92% - 97% of total financial liabilities of all financial institutions in Trinidad and Tobago between 1980 and 1984.³⁴ In a cross sectional study of corporate finance Farrell, Nijjar and Marcelle confirm the "bank-orientation" of the corporate financial market in Trinidad and Tobago with total commercial bank resources accounting for as much as 60% of total external funds and the issue of shares amounting to a mere 2.9%.³⁵ Similar results have been obtained in studies of the capital markets in Guyana and in Jamaica.

For Caribbean type economies therefore it is evident that the use of money in the production, distribution and exchange of goods and services is relatively more important than its similar use in the advanced capitalist countries and indeed its use as means of circulation for transactions in goods and services is likely to be greater than its use in financial circulation despite the considerable growth in sophistication that has occurred in the financial structure of our economies in recent years. In reconstructing the theory of money-demand we ought therefore to examine carefully the factors affecting the real demand for money.

The real demand for money was earlier defined as a demand for money destined to be employed as means of

circulation for factor income payments and purchase of intermediate inputs. Keynes was of the view that transactions in respect of factor income payments and consumption expenditure would together comprise a fairly stable proportion of the money-value of current output, while intermediate transactions were likely to change gradually in response not only to current output but to changes in the technical conditions of production and "over short periods according to whether entrepreneurs are or are not anticipating their requirement".³⁶ The price indices associated with intermediate transactions and income payments may move differently causing money expenditures on these items to behave differently from real expenditures. Additionally, transactions involving the receipt of income and expenditure out of income obey a rather different set of rules from transactions involving payment of the same income generated from production activity. But apart from observing that "one year's salary plus one year's wages plus one year's rentier interest plus one year's business earnings" will equal the annual turnover of income deposits Keynes provides no detailed explanation of the processes by which money circulates income payments as against intermediate purchases and consumption expenditure.

Marx' analysis of the circuits of capital addresses these issues frontally. Denoting money and commodities (goods and services produced for sale) by M and C respectively, the sequencing of the 'metamorphosis' of money

and commodities within the interconnected processes of production and circulation (exchange) of commodities is described by three formulae:

- I M ---> C || || C' ---> M' (Circuit of money capital)
 II || || C' ---> M' ---> C || || (Circuit of productive capital)
 III C' ---> M' ---> C || || C' (Circuit of commodity capital)

where C' and M' represent C and M augmented by surplus value while || || indicates an interruption or delay in the circuit, the duration of which is dependent on the period of production.

For Marx labour power (but not the labourer himself) is a commodity under capitalist relations of production. Thus, in formula I where money circulates in order to augment itself the first link of the formula M ---> C represents the purchase of intermediate inputs and labour power at their respective values. During the ensuing production period, || ||, surplus value is created. The final link in the circuit of money capital: C' ---> M', describes the sale of augmented C at value for M'.

In formula II the beginning and end forms of capital are quite different from those which appeared in I, but the two formulae are clearly interconnected. Formula II begins with the production process, || ||, which results in augmented C. C' ---> M' ---> C here represents the circulation of commodities - sale of augmented C or C' ---> M' and purchase of new elements of production or M' ---> C

for purposes of reproduction, $C \rightarrow M \rightarrow C$. Thus in formula II, circulation intervenes between two production processes, whereas in formula I production intervened between two phases of circulation.

In formula III commodity capital viz. augmented or produced commodities, begins and ends the circuit of capital while production interrupts it. $C' \rightarrow M' \rightarrow C$ here represents the same transactions as in formula II. However, $C \rightarrow M \rightarrow C'$, in formula III may or may not represent the same volume of transactions appearing in formula I depending on whether we assume stationary conditions (simple reproduction) or economic growth (extended reproduction).

We must now distinguish more sharply between the different rates of turnover of money as it participates in the various transactions described by the formulae. The rates of turnover of money and of commodities will depend on the duration and the number of necessary delays occurring in the circuits per unit time period. Marx distinguishes between two critical delays in the industrial circulation (i) the "time of production" and (ii) the "time of circulation". In financial circulation there are similar delays. The circuit of money is here described by the formula $M \rightarrow M'$. However, because this circuit is not associated with a specific sphere of production or of circulation but with the activities of production and of circulation in general and is, moreover, under conditions of advanced capitalism managed mainly by financial

institutions, the result is that the rate of turnover of interest-bearing capital appears to have no link to the rate of turnover of real capital described in formulae I, II and III. Yet interest-bearing capital or financial circulation is merely the converted form of "The purely technical movements performed by money in the circulation process of industrial and commercial capital".³⁷ As such, the profit earned by financial institutions is nothing but "a deduction from surplus value since they operate with already realised values (even when realised in the form of creditors' claims)".³⁸ Additionally, repeated turnover of interest-bearing capital ".... never connotes more than repeated buying and selling; while a repeated turnover of industrial capital connotes the periodicity and renovation of the entire production process (which includes the process of consumption)".³⁹ The only exception to this purely technical use of money in the financial circulation is when and in so far as capital is newly invested (or is being accumulated) in which situations ".... capital in money form appears as the starting point and the end result of the movement".⁴⁰

For Marx, therefore, the use of money in the circulation phase of the reproductive circuit viz. $C' \rightarrow M'$ or $C' \rightarrow M$, is to be associated with the use of money for purely transactions purposes, whereas in the production phase of the circuit viz. $M \rightarrow C$ || C' money is used as money capital or self-expanding value. In both phases

money is used as either (or both) means of purchase or means of payment, that is, as means of circulation. It follows that capitalist reproduction presupposes the existence of money hoards or liquidity preference. "A definite portion of the capital must be on hand constantly as a hoard, as potential money capital -- a reserve of means of purchase, a reserve of means of payment and idle capital in the form of money waiting to be put to work".⁴¹ For an individual capital the rate at which a hoard of money is turned over is dependent on the average rate of turnover of the capital itself. The average rate of turnover of the capital is inversely related to the average period of turnover (or the "time of production" plus the "time of circulation") of its component parts which in turn depends on "their different durabilities and therefore on their different times of reproduction".⁴² Given the technical conditions of production; the average rate of turnover of industrial capital will be insensitive to changes in the prices of intermediate and of final outputs and wage rates. However, the real volume of investment as well as the supply of and demand for loanable capital will be affected by these changes in prices and in wage rates.⁴³ Prices in Marx therefore act as scalars on money supply and demand in conformity with his value theory. (This closely resembles the neo-classical assumption of unit price elasticity of money demand but, as we shall see, is not its exact equivalent).

As regards the rate of turnover of money in the income expenditure circuit, Marx discusses this in terms of the rates of turnover of variable capital (wages) and of consumption out of surplus value (profits of enterprise plus rent plus interest plus other non-wage income payments) in contrast to Keynes who discusses the same subject in terms of the turnover of income deposits. Marx' analysis of consumption and savings therefore retains the linkage with his formulae of reproduction and his general analysis of the turnover of capital. Having distinguished between investments in fixed capital (machinery, plant, equipment etc.) and in circulating capital (purchases of intermediate inputs plus wages), he observes that while every turnover period will be characterized by the replacement in full of the physical elements of circulating capital, fixed capital will usually continue to exist in its old use-form except for wear and tear. The value of the annual product divided by the sum of money capital advanced for intermediate purchases and wages roughly ⁴⁴ defines the annual velocity of money invested in circulating capital. The reproduction of the capital values of intermediate inputs (in Marx, constant circulating capital) and of wage payments (in Marx, variable circulating capital) is realised in different ways, even though their rates of turnover are identical. The purchase of raw materials (and elements of fixed capital) by businessmen from each other is an exchange of capital for capital, the purchase of labour power by businessmen is on

the other hand an exchange of money for the value creating substance. The reproduction of labour power on a daily, weekly, monthly, etc. basis obeys, from the labourer's standpoint, formula III, viz. $C' \rightarrow M' \rightarrow C'$. In the first phase of this metamorphosis, $C' \rightarrow M'$, the labourer alienates his labour power which has already been "produced" by a process external to capitalist relations - although it would have been internal to a slave society. In the second phase of the metamorphosis, $M' \rightarrow C'$, the labourer reproduces his labour power by purchasing and consuming means of subsistence. ⁴⁵ (Marx generally assumes subsistence wages and a long-run savings rate of zero for workers but this assumption is not essential to the above analysis).

Now the velocity of circulation of money in the circuit of a given variable capital will in general be different from the velocities of circulation of money used to circulate that same variable capital in the income consumption circuit and the associated constant circulating capital even though the annual rates of turnover of variable capital and of constant circulating capital are identical and aggregate wage income payments add up to variable capital employed. A given variable circulating capital will in the course of a year be associated with a money velocity of circulation which is less than the money velocity associated with the same variable capital in the income consumption process unless the average interval between

purchases of intermediate inputs in any turnover period is equal to or less than the average time interval between wage receipts by workers. 46

The turnover of constant circulating capital would appear to affect the elasticity of circulation of money by way of firms' inventory policies. The higher the ratio of physical inventories of raw materials, work in progress, etc. to output the lower is the expected velocity of circulation of money associated with the turnover of constant circulating capital, and conversely, other things being equal.

It remains for us to explain the relationship between the circulation of surplus value and the circulation of money. This relationship is described by two sub-circuits:-

- (1) the circuit of surplus value consumed as revenue and
- (2) the circuit of surplus value destined for accumulation.

(2) describes the sub-circuit of savings. Money in its purely technical role as means of circulation obeys the same rules of currency regardless of the commodity being circulated by its movements. Hence, from this point of view, money in the income consumption circuit of capitalists is indistinguishable from its use by workers in the same circuit. Money destined for capital replacement or for accumulation, however, appears as latent money capital in the financial system only a very small proportion of which is at any time maintained in the form of cash or ready money. 47 The velocity of circulation of these funds, in

so far as they take the form of means of payment (credit instruments), is quite elastic depending as it does upon the degree of financial deepening, the phase of the business cycle and, especially, upon variations in interest rates. The velocity of circulation of that portion of latent money capital destined for accumulation is directly related to the rate of profit and, via the rate of profit, to the rate of turnover of fixed capital. ⁴⁸ But the average rate of interest, as we saw earlier, serves to partition gross profit between profit of enterprise (viz. industrial and commercial profit) and profit on interest bearing capital even where industrial and commercial businessmen employ internal funds exclusively. Thus, in so far as the circuit of interest bearing capital entails a money velocity of circulation which is different from that associated with the circuit of commercial and industrial capital, the real direct use of a given quantity of money by industrial and commercial capitalists will be different from its real indirect use by moneyed capitalists.

From the above discussion it can be deduced that if the theory of the demand for money is to provide us with insights into the linkage between money and production it is best to partition aggregate money demand into two compartments: (1) the real demand for money and (2) the residual demand for money. The real demand for money is now defined as the quantity of money employed by industrial and commercial businessmen in purchasing or paying for

intermediate inputs and wages or for replacing or expanding capital. Except for the inclusion of replacement investment and accumulation, this definition of the real demand for money is exactly equivalent to our earlier definition derived from Keynes' analysis.

The real demand for money will be governed by (i) the rate of turnover of circulating capital in industrial and commercial establishments, (ii) the velocity of circulation of money associated with the elements of circulating capital, (iii) the average prices of final output of establishments, (iv) the average prices of intermediate inputs, (v) average wage rates, (vi) average rates of profit and (vii) average interest rates. On the other hand, the behaviour of the residual demand for money, referring as it does to the demand for money as means of circulation in the income consumption circuit and in financial transactions, will be governed by (i) the rate of turnover of variable capital, (ii) the rate of turnover of surplus value, (iii) the proportions consumed (and saved) out of income, (iv) the velocities of circulation of money employed in transactions related to consumption and savings, (v) the average price of consumers' goods and (vi) interest rates. When average interest rates fall industrial and commercial profits (profits of enterprise) increase and the rate of return on existing money capital in industrial and commercial activity increases. The demand for money by industrial and commercial capitalists to replace investment

and to accumulate expands even as their supply of loanable capital increases.

As the supply of loanable capital to the financial system increases the demand for money by moneyed capitalists decreases, since the increase in the volume of loanable funds compensates at least in part for the decline in yield procurable from the hiring out of those funds. Conversely, when the average rate of interest rises, profits of enterprise decline, the supply of loanable funds by commercial and industrial firms declines and so does demand for money to finance replacement and new investment. Meanwhile, with the decline in the supply of loanable funds to the financial system and a fall in the rate of profit of enterprise there is certain to be an increased demand for money by the financial system in an attempt to recover previous profit positions and to compensate for the decline in the volume of loanable funds. Thus variations in the rate of interest in either direction trigger, via the real and the residual demands for money, equilibrating changes in the money market. These equilibrating changes help to stabilize average interest rates through time and, except in crises, helps to explain the rigidity in interest rate structures observed by many investigators. These equilibrating changes also provide an explanation for the instability of the interest-rate coefficient in several empirical time-series studies of the aggregate money demand

function confirming the need to partition that function along the lines suggested here.

The Model

If there are k industry groups $j = 1, 2, 3, \dots, k$, each containing a varying number of establishments numbered over $i = 1, 2, 3, \dots$, then for the ij^{th} establishment, denoting,

m_{1ij}	=	real annual demand for money
m_{2ij}	=	residual annual demand for money
w_{ij}	=	average annual wage rate paid to workers
$p_{ij}^c C_{ij}$	=	value of intermediate purchases (where p_{ij}^c and C_{ij} are price and quantity indices respectively)
n_{ij}	=	average employment per turnover of circulating capital
r	=	the annual average rate of interest on loanable capital
Π_{ij}	=	the rate of profit per turnover period
k_{ij}	=	the value of advanced capital
α_{ij}	=	the annual rate of turnover of circulating capital (constant and variable)
v_{cij}^m	=	annual velocity of circulation of money employed in the circulation of constant circulating capital
v_{wij}^m	=	annual velocity of circulation of money employed in circulating variable capital (or the wage bill per unit turnover of circulating capital)

then we may write the real annual demand for money by the ij^{th} firm as:

$$I \quad m_{lij} = \alpha_{ij} (v_{cij}^m p_{ij}^c c_{ij} + v_{wij}^m w_{ij} n_{ij}) \\ + \alpha_{ij} (\Pi_{ij} - r) k_{ij}$$

$$\text{But,} \quad \Pi_{ij} = \frac{p_{ij} q_{ij} - p_{ij}^c c_{ij} - w_{ij} n_{ij}}{k_{ij}}$$

where p_{ij} and q_{ij} are price and quantity indices respectively of output of the ij^{th} firm.

Thus,

$$I \quad m_{lij} = \alpha_{ij} (v_{cij}^m p_{ij}^c c_{ij} + v_{wij}^m w_{ij} n_{ij}) \\ + \alpha_{ij} k_{ij} \left(\frac{p_{ij} q_{ij} - p_{ij}^c c_{ij} - w_{ij} n_{ij}}{k_{ij}} - r \right)$$

whence,

$$I \quad m_{lij} = \alpha_{ij} (p_{ij} q_{ij}) + \alpha_{ij} (v_{cij}^m - 1) p_{ij}^c c_{ij} \\ + \alpha_{ij} (v_{wij}^m - 1) w_{ij} n_{ij} - \alpha_{ij} k_{ij} r$$

With α_{ij} , v_{cij}^m , v_{wij}^m each positive and typically greater than unity, the real annual demand for money by the ij^{th} commercial or industrial establishment is seen to vary directly with the value of its own output, the value of intermediate purchases and the wage bill but inversely with the rate of interest.

Estimates of within and between-industry parameters α_{ij}

v_c^m and v_w^m , may be derived from cross-sectional data

similar to, but not identical to those compiled by Bourne and Farrell, Najjar and Marcelle.⁴⁷

The residual demand function for money describes the use of money as means of circulation for the income-consumption process and for financial transactions.

Denoting the proportions of wages and of profits consumed by workers and capitalists respectively in the ij^{th} establishment by b_{wij} and $b_{\Pi ij}$, the average annual price of consumer goods consumed by workers and capitalists by $p^{w\bar{I}}$ and denoting v_{wij}^m , $v_{\Pi ij}^m$ and v_{fij}^m as the annual money velocities associated with consumption expenditure by workers, consumption expenditure by capitalists and financial transactions respectively, we may write

$$\text{II} \quad m_{2ij} = \frac{\alpha_{ij}}{p^{w\bar{I}}} \left[v_{wij}^m b_{wij} (w_{ij} n_{ij}) + v_{\Pi ij}^m (b_{\Pi ij} \Pi_{ij}) \right. \\ \left. + v_{fij}^m r \left\{ (1 - b_{wij}) w_{ij} n_{ij} + (1 - b_{\Pi ij}) \Pi_{ij} \right\} \right]$$

The data required for estimating equation II at the level of the firm does not exist. However by summing across firms and industries and denoting Y_W and Y_{Π} as annual wage and non-wage incomes respectively generated by commercial and industrial establishments we may write the aggregate annual residual demand for money as

$$II' M_2 = \frac{\alpha}{p^{wII}} \left[v_w^m b_w Y_w + v_{II}^m b_{II} Y_{II} + v_f^m r (S_w Y_w + S_{II} Y_{II}) \right]$$

where α , v_w^m , v_{II}^m , v_f^m , b_w and b_{II} are to be interpreted as weighted averages of the establishment counterparts in equation II, and S_w , S_{II} are the proportions saved out of annual wage and non-wage incomes respectively. Further manipulation of II' yields

$$II' M_2 = \frac{\alpha}{p^{wII}} \left[\left(v_w^m b_w + v_f^m r - v_f^m r b_w \right) Y_w + \left(v_{II}^m b_{II} + v_f^m r - v_f^m r b_w \right) Y_{II} \right]$$

from which it is seen that M_2 varies directly with Y_w and Y_{II} , provided, ⁽⁵⁷⁾

$$0 < \frac{v_w^m}{v_f^m} + \frac{r}{b_w} > r$$

$$0 < \frac{v_{II}^m b_{II}}{v_f^m b_w} + \frac{r}{b_w} > r$$

It is evident that several of the parameters appearing in equations II and II' are linked with those appearing in equation I. Estimation of the coefficients of II' therefore may be derived from either (both) cross-sectional or (and) time-series data obtained from the national income accounts.

Summary and Conclusions

The theory of money demand plays a critical role in modern macro-economics. However, despite considerable theoretical and empirical research on the money-demand function it remains ill-defined. In this paper we have sought to reconstruct that theory on micro-theoretical foundations rather different from those to be found in the Keynesian and neo-classical versions. Marx' analysis of the circuits of capital in a modern capitalist economy has helped us to distinguish two kinds of money demand: (1) the real demand for money by industrial and commercial establishments and (2) the residual demand for money for income-consumption and financial transactions. The real demand for money is associated with the use of money in transactions involving (i) purchases of intermediate inputs, (ii) payments for labour services and (iii) replacement investment and accumulation.

Our analysis shows that the real demand for money varies directly with the prices and quantities of final output and intermediate inputs and with the wage bill of firms, but it is inversely associated with the rate of interest. The residual demand for money on the other hand varies directly with wages, profits and the rate of interest but inversely with consumer prices.

Real Demand
 $m_1 = f(P, Y, W, r)$
 $f_P > 0, f_Y > 0, f_W > 0, f_r < 0$

Residual Demand
 $m_2 = g(W, \pi, r, P)$
 $g_W > 0, g_\pi > 0, g_r > 0, g_P < 0$

We expect the real demand for money to be relatively more important in Caribbean economies than it is in the advanced capitalist countries. Tests of the model of money

demand presented in this paper will however require the use of cross-sectional survey data as well as time-series data from the national accounts.

Unlike neo-classical and even Keynesian models of money demand, the model presented here does not assume stability of the coefficients associated with the aggregate representation of the variables appearing in the macro-functions since each of these coefficients will be more or less complex linear combinations of structural parameters. These structural parameters are a set of velocities, turnover rates for circulating and fixed capital, consumption and savings proportions. Our theory focuses on the identifiability and the stability of the structural parameters themselves. These characteristics of the parameters will depend upon changes in technical conditions of production and the phase of the business cycle among other factors. Among the technical conditions of production are to be included (a) the rate and pattern of accumulation in industry and (b) the composition of capital especially as it affects the capital/labour ratio and labour productivity. Estimates of the structural parameters entering the real demand for money function are essential to an understanding of the influences exerted by these factors.

The residual demand for money function linking as it does money with its use as means of circulating expenditures out of income illustrates, in our analysis of the circuits of capital, the linkages between production, consumption and

the money market. The rate of interest plays opposite roles in our theory.

An increase (decrease) in the rate of interest increases (decreases) the residual demand for money but decreases (increases) the real demand for money. This is because an increase (decrease) in the rate of interest reduces (increases) the average profit of commercial and industrial firms but simultaneously reduces the supply of loanable capital to financial institutions. We have suggested that these dual effects of interest rate variation help to explain the observed stability in interest rate structures through time but they also explain the volatility of the interest rate coefficient often generated from time-series studies of the aggregate money demand function. It follows from our theory that an increase in money supply accompanied by a fall in the rate of interest will increase the real demand for money but reduce the residual demand. The break on the further fall of interest rates created by these opposing tendencies combined with the build-up of the supply of loanable capital in the financial system send incorrect signals to the actors in the money market and provide fertile ground for a growth in turnover of financial transactions unrelated to the industrial circulation. Similarly, a fall in money supply accompanied by an increase in average interest rates sets in motion opposite tendencies within our model resulting in a limitation on the rise of interest rates and a reduction in the real demand for money

even as the residual demand for money increases in an attempt to compensate for the reduced supply of loanable funds. These tensions in the money market may erupt in a crisis as the actors misread the indicators of real money demand.

Our reconstruction of the theory of the demand for money also shows that the assumption of unit price elasticity of money demand is generally inappropriate. The real demand for money varies directly with the average prices of final output and of intermediate inputs while the residual demand for money varies inversely with the average price of consumer goods. These different price effects are no doubt related to the fact that while commercial and industrial establishments are in a position to pass on to consumers some portion at least of the increase in prices of the elements of circulating capital, in the residual demand for money function there is limited possibility for such price shifting by consumers and by participants in the money market. Moreover, income either in the form of wages or surplus value (non-wage income) will generally need to be earned before it is available for spending; if prices of consumer goods rise in the interim the residual demand for money can only be increased if there are appropriate changes in money velocities or consumption (savings) proportions or in the rate of interest. If these structural parameters remain unchanged, the residual demand for money will vary inversely with the average price of consumer goods.

Finally, the theory of money demand presented in this paper clarifies the so-called "competing asset" effect between money and other capital assets. This effect is seen to reside only in the real demand for money-function where the rate of interest variable carries a negative coefficient because of its role there in partitioning the surplus of establishments into profit of enterprise and interest on money-dealing capital. But in the residual demand function for money, the positive coefficients on the interest rate variable indicate that in the financial circulation money is complementary with other financial assets because the demand for loanable funds by the financial system varies directly with the average rate of interest.

The distinction made in this paper between the real and the residual demands for money vividly illustrates not only the money-production transmission mechanism but the processes of the money-market and the contradictory effects of interest-rate variation.

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4. On techniques for determining statistical causality in time-series analyses see, for example, C.W.J. Granger, "Investigating Causal Relations by Econometric Models and Cross-Spectral Methods", *Econometrica*, Vol. 37, No. 3, July 1969, pp. 424-438 and Christopher A. Sims, "Money, Income and Causality", *American Economic Review*, Vol. LXII, 1972, pp. 540-552.
5. See E.B.A. St. Cyr, "A Note on the Trinidad and Tobago Inflationary Experience, 1965-1976", *Social and Economic Studies*, Vol. 28, No. 3, September 1979 and Wendell A. McLean, "Some Evidence on the Demand for Money in a Small Open Economy: Barbados", *Social and Economic Studies*, Vol. 31, No. 3, 1982.
6. Bourne, however, notes the following features which he considers to be peculiar to the Caribbean-type economy, (i) the restricted range of financial assets; (ii) relative rigidity of interest rates and (iii) uncertainty in the minds of households and firms concerning the future values of wage rates, commodity prices and (less directly) import prices and the exchange rate. Characteristic (i) is undoubtedly a feature which could modify both the neo-classical and Keynesian type money demand functions but (ii) and (iii) appear to be less capable of doing so. In any event it is interesting to note that Bourne's empirical money demand functions are essentially truncated dynamic versions of the typical money demand function of neo-classical theory. (cf. Milton Friedman, *The Optimum Quantity of Money and Other Essays*, 1969, p. 58).
7. J.M. Keynes, *A Treatise on Money*, Vol. I, (Macmillan), 1965, Chapter 3, pp. 34-49.

8. *Ibid.*, p. 43.
9. J.M. Keynes, *The General Theory of Employment, Interest and Money*, (Macmillan), 1960, p.107.
10. Milton Friedman, *op.cit.*, p. 54.
11. David Laidler, *The Demand for Money: Theories and Evidence*, 1969, p. 118.
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18. See Meghnad Desai, *Testing Monetarism* (London), 1981, Chapters 3 and 4.
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21. J.M. Keynes, *A Treatise on Money*, Vol. I (Macmillan), 1965, Chapter 3, p. 46.

22. J.M. Keynes, *The General Theory of Employment, Interest and Money*, p. 168.
23. *Ibid.*, pp. 199-202.
24. Karl Marx, *Capital*, Vol. II, (International Publishers, New York), 1967, pp. 23-120.
25. *Capital*, Vol. II, p. 362 foll. and Karl Marx, *Theories of Surplus Value* (Volume IV of *Capital*), Part I (Progress Publishers), 1968, pp. 97-151.
26. It is this weakness of the transaction velocity concept that has, we think, motivated Wendell McLean's use of an augmented proxy for transactions in the case of Barbados. The small-country argument he has employed to justify the use of income plus imports as the scale variable in his money-demand function is more correctly interpreted as a method of allowing for transactions in pre-existing capital goods which are mostly imported by Barbadian businessmen. Note further that in Marx' writings pre-existing capital goods refer to fixed capital and intermediate capital goods so that the definition covers a wider set of commodities than is described by the inter-industry sub-matrix of an input-output table.
27. J.M. Keynes, *A Treatise on Money*, Vol. I (Macmillan), 1965, p. 248, also Karl Marx, *Capital*, Vol. III, Chapters XXV and XXVII.
28. J.M. Keynes, *A Treatise on Money*, Vol. I, pp. 250-257.
29. Karl Marx, *Capital*, Vol. III, pp. 463-464.
30. J.M. Keynes, *ibid.*; Karl Marx, *Capital*, Vol. III, pp. 484-485.
31. J.M. Keynes, *A Treatise on Money*, Vol. I (Macmillan), p. 48.
32. "If we enquire... why the limits of a mean rate of interest cannot be deduced from general laws, we find the answer lies simply in the nature of interest. It is merely a part of the average profit. The same capital appears in two roles - as loanable capital in the lender's hands and as industrial or commercial capital in the hands of the functioning capitalist. But it functions just once, and produces profits just once.
- In the production process itself the nature of capital as loanable capital plays no role. How the two parties who have claim to it divide the profit is in itself just as purely empirical a matter belonging to

the realm of accident as the distribution of percentage shares of a common profit in a business partnership. Two entirely different elements - labour power and capital - act as determinants in the division between surplus value and wages, which division essentially determines the rate of profit; these are functions of two independent variables, which limit one another; and it is their qualitative difference that is the source of the quantitative division of the produced value... the same occurs in the splitting of surplus value into rent and profit. Nothing of the kind occurs in the case of interest. Here the qualitative differentiation... proceeds rather from the purely quantitative division of the same surplus value.

It follows... that there is no such thing as a natural rate of interest. But... the rate of interest appears as a uniform, definite and tangible magnitude... In so far as the rate of interest is determined by the rate of profit, this is always the general rate of profit and not any specific rate of profit prevailing in some particular branch of industry, and still less any extra profit which an individual capitalist may make in a particular sphere of business". *Capital*, Vol. III, pp. 364-365.

33. Compton Bourne, "Financial Deepening, Domestic Resource Mobilization and Economic Growth: 1953-1981", in *Money and Finance*, Bourne and Ramsaran, *op. cit.*, p. 242.
34. Bourne and Ramsaran, *op. cit.*, pp. 8-9.
35. Terence Farrell, Annette Najjar and Hazel Marcelle, "Corporate Financing and Use of Bank Credit in Trinidad and Tobago", *Social and Economic Studies*, Vol. 35, No. 4, 1956, pp. 25-26.
36. J.M. Keynes, *A Treatise on Money*, Vol. I (Macmillan), p. 47.
37. *Capital*, Vol. III, p. 315.
38. *Ibid.*, p. 322.
39. *Ibid.*, p. 303.
40. *Ibid.*, p. 315.
41. *Ibid.*, p. 316.
42. *Capital*, Vol. II, p. 183.
43. *Capital*, Vol. II, pp. 285-292.

44. Roughly because we must make allowance for wear and tear (depreciation) already included in the value of final output. Further, stationary conditions are here assumed e.g., all prices remain unchanged, all output is sold, the entire circulating capital is consumed etc. Under these conditions an annual rate of turnover of circulating capital of 10 would be equivalent to an average turnover period of 1 1/5 months.

45. Surplus value and profits arise in the exchange of labour-power for capital, according to Marx, because labour-power is the sole commodity capable of creating a quantum of value in excess of its own value.

46. The underlying assumption here is that while businessmen may hold inventories of raw and auxiliary materials they are unable to do so in the case of labour-power. Consequently, the frequency of purchases of the former will tend to be less than that of the latter during any turnover period. This general rule is, of course, modified by trade union contracts, the nature of the raw materials, payments practices and customs etc.

The money-velocities associated with each of these two elements of circulating capital are nevertheless expected to be different.

47. *Capital*, Vol. II, p. 349.

48. *Capital*, Vol. II, p. 492 foll.

49. See footnote 20.

50. With $0 < b_w$, $b_f < 1$ and v_w^m , v_f^m , v_{II}^m each positive and greater than one, these conditions are assured.