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AN ALTERNATIVE APPROACH FOR THE ANALYSIS AND FORECASTING OF ECONOMIC SERIES: THE STATE SPACE MODELLING

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An alternative approach for the analysis and forecasting of economic series : the state space modelling

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The infatuation for econometrics research at the end of the 80's has led today to a wide diversity of concepts for the description and the analysis of economic variables as well as to a wide heterogeneity of the methods for the modelling and forecasting of theses variables.

Firstly, in consequence of the criticism of Sargent (1979) and Sims (1980), VAR models have taken quite a significant position in some empirical studies. The causality tests and the impulse responses functions have been widely used in order to test some theories, not only for precise questions such as the correlation between money and activity but also for more general questions such as the specification of macroeconomic models. Motivated by the necessity of taking into account the non stationarity of time series, the economists have then dedicated an abundant literature to integration and cointegration tests. Now, these tests are an ineluctable stage to describe the individual and joint evolutions of variables. The ARCH model which was also introduced in this decade by Engle [1982] gives the possibility to take directly into account the temporal variation of the variance of the studied series.

Among all the developments made recently, there is also a renewal of interest for the state space modelling. Indeed, after the successes of the famous Kalman filter in the numerous applications of the systems theory, Akaike [1974] and Aoki [1976] had already suggested to apply this approach to the study and the forecast of economic series. But the majority of the economists did not adopt it. It seemed to be difficult to apply because of the choice of the parameters of Kalman filter but also because of the calculation formulas.

This change of opinion about the use of the state space framework for the study of the economic series is the result of the new methodology proposed by Aoki [1987,a]. Based on the notion of "balanced realization", it aims to build a state model directly from studied data. In this respect, this approach is bound to be much more used than it is nowadays.This is all the more right since it suits as well the forecasting of univaried series as the analysis of joint dynamic of several series. On this point, empirical studies of recent econometric literature bring illustrations of the wide diversity of the application field of the state space approach. For univariate series, Kohn and Ansley [1985,1986] elaborated algorithms of estimation, interpolation and forecasting for series which might have missing data. Shea [1987,1989] proposed a procedure that allows the calculation of the likelihood function of an ARMA process from the state space formalization. As another example, Barone [1987] elaborated a method that allows to generate stationary multivariable gaussian series relying on a state representation. Concerning multivariate series, applications are also numerous. Here are two examples only. Aoki [1987,b] studied the interdependence between economic variables of Japan and the USA and showed that state space modelling allows a more accurate

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interpretation and analysis as compared with VAR models. Vinod et Basu [1995] also estimated a state model for consumption, income and interest rate from american data and use it to have a different point of view upon the empirical investigations of the actual business cycle theory.

Considering the quality of the results it has already provided, it is important to emphasize the fact that state space modelling offers serious perspectives. Besides, considering its properties, we share the idea that this approach is a good option for the study and forecasting of macroeconomic series. Indeed, the study of evolution of variables and the determination of optimal economic policies can be undertaken in a rich methodological surrounding based on the concepts of systems theory and which allows the identification and specification of a model, the analysis of the dynamic properties of this model, the elaboration of optimal control laws and simulation (Pagan and Preston (1982), Maurin (1993)). In the same way, if we only stick to the fact that state space modelling are asymptotically stable (Aoki (1987,b)), we can suppose that their results in forecasting can be stronger than the ones supplied by VAR models.

Initiated at the beginning of the 70's, the work dedicated to macroeconometric modelling in the Caribbean have led today to the existence of a rather wide range of models, but as Craigwell et al (1995) stress it, many of these models present some deficiencies regarding their theoretical specification. If we are to make an assessment of their concrete use, it is suitable to put forward the fact that none of them was actually useful either to the preparation of the economic policy or to the calculation of forecasts.

Since the data of several key variables are missing or are available on relatively short periods, and only in annual periodicity, it is obvious that the deficiencies of the statistical apparatus in the Caribbean countries constitute a major explanation for this rather dark report.

In this context, it is useful to search for some alternatives and to ponder over the question : what could other modelling approaches bring ? The aim of this work is therefore to examine what could be the contributions of the state space representation to modelize and forecast the evolution of caribbean economic variables.

Our paper is organized as follows. The first section gets on the problem of identification of a space structure directly from observed data and the estimation of the its parameters. In the second section, the implementation of the method is illustrated with a particular emphasis on its interests for the treatment of caribbean series.

1. Realization of balanced models and forecasting of economic series

The minimization of a quadratic criterion under the constraint of a linear system and the estimation by Kalman filter have been till the end of the 80's the categories of problems of the control theory which gave rise to many applications in economics. Now, since the formalization of Aoki (1987) which aims at leading to a state model directly from observed data, the identification of a multivariate system represents another important field of the control theory to be the subject of economic applications.

In the context of the systems theory, the identification of a system consists in determining a state model, that is to say a quadruplet (A_t, B_t, C_t, D_t) of minimal size which has to be compatible with the known data of the input and output variables. Before the 80's, one distinguished three basic approaches : the methods resulting from least squares, the maximum likelihood and the instrumental matrix method (see Doncarli et Larminat (1978) for a survey). These last years, the methods deriving from the concept of "balanced realization" introduced by Moore (1981) asserted themselves (see for example Verhaegen and Depprettere (1991), Van Der Klaw et al. (1991), Ljung (1991)).

For the economists, Aoki (1987) was the first to bring his contribution to the problem of the identification of an economic system. However, Akaike (1974) is the one who introduced a state space parametrization for modelling time series. Close to the one which is used in the Kalman filter, this parametrization has the drawback to lead to different and non equivalent models when the size of the state vector is modified. From writing a state model, so-called innovation form and relying on the balanced realization technique, Aoki established a procedure which supplies a single state representation of minimal size. This procedure relies on the singular value decomposition of a Hankel matrix built from the autocovariance function of each series.

After Aoki, the researches led to different methods. For example, Otter and Van Dal (1987) formulated a variant which uses a Hankel matrix defined from the covariances between the innovations and the studied data. Mittnik (1989) (see appendix 2) also studied several identification schemes from a state representation in which the state vector is brought up to date from the output observed at instant t instead of the innovation of this same instant. As another variant, Havenner and Criddle (1987) proposed a procedure which relies on a Hankel matrix built from centred and reduced data.

Even if these methods offer great particularities, they all articulate themselves in two stages : obtaining a Hankel matrix thanks to an adequate technique on the one hand, and calculating of the parameter of the model by applying the results of system theory on the other hand..

1.1 The procedure for parameter estimation

So let $\{y_t; t = 1, \ldots, N\}$ be a set of centered and stationary observations of a y vector which regroups q variables representating the evolution of an economic phenomenon observed at the instants $t=1,\ldots, N$ and let the innovation form[2] be :

$$\begin{cases} z_{t+1} = Az_t + Ge_t \\ y_t = Cz_t + e_t \end{cases} \tag{4}$$

[2] The innovation form stands out by the presence of the same innovations in the state equation and the observation equation. The choice of this model does not lose generality because the passage to the classical model is done by a spectral factorization of the matrixes (A, G, C).

where the innovations e_t are both serially independant with covariances Δ_e and independant of state variables z_t ($Cov(z_t, e_l) = 0; \forall t, l$).

The usual procedure to determine the parameters (A, G, C) which "realize" the system (4) is composed of the following stages.

(i) construction of the Hankel matrix

Let us note $\Delta_k = Cov(y_t y_{t+k})$ the covariance matrix between y_t and y_{t+k}. We have :

$$E\{y_t y'_{t+k}\} = E\{(Cx_t + e_t)(Cx_{t+k} + e_{t+k})'\}$$
$$= CE\{x_t x'_{t+k}\}C' E\{x_t e'_{t+k}\} + E\{e_t x'_{t+k}\}C' + E\{e_t e'_{t+k}\}$$

Since $E\{e_t x'_{t+k}\} = 0 \forall t$, and that the e_t are not autocorrelated, we get :

$$\Delta_k = \begin{cases} C\Pi C' + \Delta_e & \text{if } k = 0 \\ CA^k \Pi C' + CA^{k-1} G\Delta_e & \text{for } k = 1,2,\ldots t \end{cases} \quad (5)$$

with $\Pi = E\{x_t, x'_t\}$ and $\Delta_e = E\{e_t, e'_t\}$

Indeed, when $k = 0$ the relation above is obvious. For $k \geq 1$, we develop the transition equation of the model (4). We obtain

$$x_{t+k} = A^k x_t + \sum_{j=0}^{k-1} A^{k-j-1} G e_{t+j} \quad (6)$$

which allows us to write :

$$E\{x_t, x'_{t+k}\} = A^k E\{x_t, x'_t\} = a^k \Pi \quad (7)$$

By puting down $s = t + k$, (6) becomes :

$$x_s = A^k x_{s-k} + \sum_{j=0}^{k-1} A^{k-j-1} G e_{s-k+j}$$

We deduce $\quad E\{x_t, e'_{t+k}\} = A^{k-1} G$

Generally, one use an Hankel's hypermatrix to represent a finite or unfinite series of matrixes. With the autocovariance function $\{\Delta_k\}$, we get this hypermatrix H in the following way :

$$H = \begin{bmatrix} \Delta_1 & \Delta_2 & \Delta_3 & \Delta_4 & \cdots \\ \Delta_2 & \Delta_3 & \Delta_4 & . & \cdots \\ \Delta_3 & \Delta_4 & \Delta_5 & . & \cdots \\ \Delta_4 & . & \cdots & . & \\ . & . & \cdots & . & \\ . & . & \cdots & . & \\ . & . & \cdots & . & \end{bmatrix} \qquad (8)$$

From the observations $\{y_t; t = 1,\ldots,N\}$, we can calculate an approximation $\hat{H}$ of H in many different ways. The one wich is directly deducted from the autocovariance matrixes is defined by :

$$\hat{H} = \begin{bmatrix} \hat{\Delta}_1 & \hat{\Delta}_2 & \cdots & \hat{\Delta}_r \\ \hat{\Delta}_2 & \hat{\Delta}_3 & \cdots & . \\ . & . & \cdots & . \\ . & . & \cdots & . \\ . & . & \cdots & . \\ \hat{\Delta}_f & \hat{\Delta}_{f+1} & \cdots & \hat{\Delta}_{f+r-1} \end{bmatrix} \qquad (9)$$

r and f being respectively the number of block-columns and the number of block-lines by columns, and $\hat{\Delta}_i$ an estimator of Δ_i.

Because of the stationarity of the process $\{y_t\}$, $\hat{H}$ coincides with the estimator of the matrix

$$E\left\{ \begin{bmatrix} y_{t+1} \\ y_{t+2} \\ . \\ . \\ . \\ y_{t+f} \end{bmatrix} \left[y'_t \; y'_{t-1} \cdots y'_{t-r+1} \right] \right\},$$

that is to say the autocovariance matrix between observed values of the variables y_t and the future realisations of these variables $(y_{t+1},\cdots,y_{t+f})$, calculated from the present and past observations of y. In that case, r indicates the maximum lag to represent the memory of the process and f is an integer wich depends on the horizon of prediction h $(f \geq \max(pr,h))$.

An unbiased estimator of Δ_i is $\hat{\Delta}_i = \frac{1}{N-i}\sum_{t=1}^{N-i} y_{t+i}y_t, i = 0,1,\ldots,r+f$.

However, it is advisable to choose the quantity $\hat{\Delta}_i = \frac{1}{N}\sum_{t=1}^{N-i} y_{t+i}y_t$ which corresponds to the maximum likelihood estimator of Δ_i.

(ii) Choice of the model order

By definition, the state vector is interpreted as the memory of the system, it synthetizes all the information on its past evolution. In relation to the particular structure of the Hankel matrix, this information corresponds in some respects to relationship measures between the past and the future of the variables. It is therefore particularly suitable for the forecasting of the future values of y_t from its past observations. We then understand that the quality of adjustment of a state model from observed data of y_t and the quality of the forecasts y_{t+i}, $i = 1,2,...$ depend on the chosen value for the number of components of the state vector.

This choice is made firstly by deciding the values to give to the parameters r and f. Great values entail a minimal loss of information in the approximation of H by $\hat{H}$ but, in return, generate bigger errors. Then, the number of states n that can synthetize the information contained in $\hat{H}$ is given by the rank of this matrix defined as the number of nonzero singular values of $\hat{H}$ (Kronecker theorem).

(iii) Calculation of the matrices $\hat{A}$, $\hat{G}$ and $\hat{C}$

We calculate the estimators $\hat{A}$, $\hat{G}$ and $\hat{C}$ by performing two factorizations of the matrix $\hat{H}$. On the one hand,

The structure of the matrixes Δ_k defined in terms of the parameters of the model (4) and of the covariances Π et Δ_k enables the matrix H to have the following remarkable property :

$$H = \begin{bmatrix} C \\ CA \\ \cdot \\ \cdot \\ \cdot \\ CA^{f-1} \end{bmatrix} \begin{bmatrix} \Omega & A\Omega & \cdot & \cdot & \cdot & A^{r-1}\Omega \end{bmatrix} \qquad (10)$$

H is writen as the product of the observability matrix by a matrix whose form is similar to that of the commandability matrix of system (4)
From relations (5) and (10), we can establish the expression of Ω :

$$\Omega = A\Pi C' + G\Delta_e \qquad (11)$$

Since C, A and Ω are unknown and that on the other hand H can be estimated, we deduce from its singular values decomposition :

$$\hat{H} = \hat{\vartheta}\hat{\mathcal{C}} = \hat{U}\hat{\Sigma}\hat{V} \qquad (12)$$

Equality which permits then several factorizations for $\hat{\vartheta}$ and $\hat{\mathcal{C}}$. We can have for example $\hat{\vartheta} = \hat{U}'$ and $\hat{\mathcal{C}} = \hat{\Sigma}\hat{V}'$ or $\hat{\vartheta} = \hat{U}'\hat{\Sigma}^{1/4}$ and $\hat{\mathcal{C}} = \hat{\Sigma}^{3/4}\hat{V}'$. Among these factorizations, Moore (1981) showed that the most interesting is the one which implies the equality of the gramians

$$\hat{\vartheta}'\hat{\vartheta} = \hat{\mathcal{C}}\hat{\mathcal{C}}' = \Sigma \qquad (13)$$

and the obtention of these last as the unique solution of Lyapunov[3] equations.

$$A'XA - X = -C'C \qquad (14)$$
$$AXA' - X = -\Omega\Omega'$$

It is defined by the relations (15) below and comes down to carrying out a change of basis in the state space to obtain a so called balanced representation :

$$\hat{\vartheta} = \hat{U}'\hat{\Sigma}^{1/2} \qquad \hat{\mathcal{C}} = \hat{\Sigma}^{1/2}\hat{V}' \qquad (15)$$

With this decomposition, the estimators of the parameters are obtained in two stages. Firstly, we show that $\hat{A}$, $\hat{\Omega}$ and $\hat{C}$ are given by :

$$\hat{A} = \Sigma^{-\frac{1}{2}}U'\bar{H}V\Sigma^{-\frac{1}{2}}$$
$$\hat{\Omega} = \Sigma^{-\frac{1}{2}}U'H^{\Omega} \qquad (16)$$
$$\hat{C} = H^{C}V\Sigma^{-\frac{1}{2}}$$

the matrices H^{C} , H^{Ω} and $\bar{H}$ being defined as follows :

$$H^{C} = \begin{bmatrix} \hat{\Delta}_1 & \hat{\Delta}_2 & \cdots & \hat{\Delta}_r \end{bmatrix} \qquad (17)$$

$$H^{\Omega} = \begin{bmatrix} \hat{\Delta}'_1 & \hat{\Delta}'_2 & \cdots & \hat{\Delta}'_f \end{bmatrix} \qquad (18)$$

$$\bar{H} = \begin{bmatrix} \hat{\Delta}_2 & \hat{\Delta}_3 & \cdots & \hat{\Delta}_{r+1} \\ \hat{\Delta}_3 & \hat{\Delta}_4 & \cdots & \hat{\Delta}_{r+2} \\ . & . & \cdots & . \\ . & . & \cdots & . \\ . & . & \cdots & . \\ \hat{\Delta}_{f+1} & \hat{\Delta}_{f+2} & \cdots & \hat{\Delta}_{f+r} \end{bmatrix} \qquad (19)$$

Secondly, we calculate $\hat{G}$, using the matrix Π, the solution of the equation (20) obtained from (1):

$$\Pi = A\Pi A' + G\Delta_e G' \qquad (20)$$

From relations (5) and the definition of Ω we obtain the following Riccati equations which ensure the stationarity of the process $\{x_t\}$.

$$\Pi = A\Pi A' + (\Omega - A\Pi C')(\Delta_0 - C\Pi C')^{-1}(\Omega - A\Pi C')' \qquad (21)$$

Its resolution may be apprehended by an iterative algorithm which initialyzes the matrix Π to the zero matrix and which achieves its updating directly from relation (21). Another process is

[3] In the systems theory, the model (4) is said stable if and only if, for a positive matrix Q, there is a positive matrix P, unique solution of the Lyapunov equation $A'PA - P = -Q$.

based on a non-iterative algorithm similar to the one introduced by Laub (1983). It proceeds within two stages. First the construction of a symplectic matrix from the estimated parameters, then the calculation of the solution from the transformation of this matrix into the real Schur decomposition form (Aoki (1987,a)).

Once this solution is found, the estimator of $\hat{G}$ is deducted from (11) :

$$\hat{G} = (\Omega - A\Pi C')(\Delta_0 - C\Pi C')^{-1} \tag{22}$$

1.2 Properties of the estimated model

The principle of the procedure outlined above consists in extracting the essential of the information contained in the Hankel matrix by approximating the space spaned by this matrix by a space of lower dimension spaned by the singular vectors associated to the nonzero singular values. Taking advantage of the concepts and tools of the systems theory, it naturally provides a model with salient properties which are of two orders.

First, specific properties stemming directly from the coordinate system and the algebraic transformations chosen to project the initial variables in the state space.

- The strict nestedness and the orthogonality of the models associated to the subspaces included into the state space. The states are chosen in such a way so that any model $(\hat{A}^*, \hat{G}^*, \hat{C}^*)$ of dimension n^* less than n gets directly nested inside the models of superior size. Therefore, $(\hat{A}^*, \hat{G}^*, \hat{C}^*)$ is obtained straight away by extraction of the n^* first columns of $\hat{A}$, $\hat{G}$ et $\hat{C}$.

- The parametrization defined by the transformation (16)-(22) enables to identify in a unique way the matrices $\hat{A}$, $\hat{G}$ et $\hat{C}$. It ends up into a state vector whose components are completely reachable and observable. This state vector is also of minimum size, the space associated to it is the same as the smallest subspace spaned by all the past and future observations of the system output.

- $\hat{G}$ is the gain matrix of the Kalman filter associated to model (4). It is well known that this filter provides the optimal estimation of the state variables which are not observable but whose knowledge is prior to the calculation of forecasts of the variables y_t (see Anderson and Moore (1979)). In this filter, it is the coefficients of the matrix $\hat{G}$ which determine the weighting to attribute to the forecasts errors which occur in the calculation of the best prediction of state variables :

$$\hat{x}_{t+1|t} = \hat{A}\hat{x}_{t|t-1} + \hat{G}_t(y_t - \hat{C}\hat{x}_{t|t-1}) \tag{23}$$

Beside those properties of stability and minimality, it is advisable to notice that the technics involved in this procedure lead to a model which enjoys some general properties very useful to describe and forecast correctly the dynamic behavior of the variables.

- The model is asymptotically stable if the initial series are weakly stationary. As we showed it previously, this result is the direct consequence of restrictions imposed on the form of the observability and commandability matrices $\hat{\mathcal{O}}$ and $\hat{\mathcal{C}}$. Such a property is not guaranteed by the VAR methodology.

- We can consider that most of forecasting models, for univariate as well as multivariate series, constitute particular cases of state space models.

- Because the Kalman filter is the most effective among the adaptative estimation algorithms, the state space models have the flexibility to deal directly with gross data of time series and, to take account the non-stationarity arising from trend and seasonality.

To finish with the discussion on the characteristics of the procedure, we won't forget to mention that other fields of investigation on the possibilities of state space models deserve some attention. To give just an example, we think that it is possible to elaborate a variant of Aoki procedure which would provide a cointegration test. The justification of this comes from the fact that the number of state n, obtained by selection of the nonzero singular values, is equal to the number of cointegration relations between the components of y_t. A definite interest of such a test would be to suggest an alternative to the Johansen and Juselius (1990) procedure. This is all the more right since we know that on the numerical level, is it better to formulate a rule of decision on the basis of singular values rather than on the basis of eigenvalues.

1.3 Forecasts computation

Once the realization $(\hat{A},\hat{G},\hat{C})$ is known, one can predict the values of the series $\{y_t\}$ in many different ways. We note $\hat{y}_{t+h|t}$ the forecast performed on the date t for the horizon h.

A first method consists in obtaining the $\hat{y}_{t+h|t}$ by solving the system (4) from an estimation of initial state x_1. The latter can be obtained, according to the backcasting tehnique of Box and Jenkins which is an iterative process calculating successively backwards forecasts and forwards forecasts. On the basis of the recurrence

$$\begin{cases} \hat{x}_{t+1} = \hat{A}\hat{x}_t + \hat{G}\hat{e}_t \\ \hat{e}_t = y_t - \hat{C}\hat{x}_t \;, \quad t = 1,\dots,N-1 \\ \hat{x}_0 = 0 \end{cases} \tag{24}$$

we obtain firstly an estimation of x_T by looking further back in time. Then from this estimation $\hat{x}_T$, we calculate forecasts backwards till we get to $\hat{x}_1$ whose value is given by

$$\hat{x}_1 = \hat{G}\hat{e}_t + \sum_{i=1}^{N-1}(-1)^i \hat{A}^i \hat{G}\hat{e}_{1+i} \tag{25}$$

Another method proceed by conversion of the model (4) to an ARMA model. Indeed, since the state of the system x_t on the date t corresponds to the best prediction in $t-1$ of the Kalman filter, we can write by eliminating e_t

$$x_{t+1|t} = (\hat{A} - \hat{G}\hat{C})x_{t|t-1} + \hat{G}y_t$$

$$y_{t+1|t} = \hat{C}\hat{G}y_t + \hat{C}(\hat{A} - \hat{G}\hat{C})x_{t|t-1}$$

By developping the expression of $x_{t-k|t-k-1}$ we obtain :

$$y_{t+1|t} = \hat{C}\hat{G}y_t + \ldots + \hat{C}(\hat{A} - \hat{G}\hat{C})^k \hat{G}y_{t-k} + \hat{C}(\hat{A} - \hat{G}\hat{C})^{k+1} x_{t-k|t-k-1} \quad , k=1,2,\ldots \qquad (26)$$

The elimination of $x_{t-k|t-k-1}$ therefore leads to a recurrence relation linkink the forecast $y_{t+1|t}$ only to the present and past observations of y_t. It can be done by means of the Caley-Hamilton theorem[4].

1.4 Computer implementation

With regard to their properties and links with other families of models. it appears clearly that state space models offer various technics to establish forecasts.

Paradoxically, these technics are still absent from the main econometric software for the treatment of times series. It is the case for RATS (windows version), micro-TSP (version 7), SORITEC, SAS PC Forecast, BMDP/PC and FORECAST MASTER. If some of them propose a few commands for the modelling by state variables or the application of Kalman filter, the fact remains that the standard procedure of identification and estimation (Aoki's method) as well as its variants (Mittnik, Havenner and Criddle, ...) are not available yet within these software.

In this respect, the perfecting of a tool which would contribute to a wider use of state space models for the analysis and the forecast of economic variables, seems to be a pressing necessity.

In our opinion, two ways can be considered to reach this objective. Following the example of the CATS program of Johansen et Juselius (1992) for cointegration tests, a first solution would consist in developping a "state space module" within an econometric software. We could also conceive and write in a programming language, a software especially dedicated to the application of the state space modelling concepts. Whatever the case, resorting to programming reveals itself to be unavoidable for the one who would like to use the state space models.

[4] If $P(\lambda) = \lambda^n - a_1\lambda^{n-1} - \ldots - a_n$ is the characteristic polynomial of the matrix A, then $P(A)=0$ and any power of A, A^i with $i \geq n$, can be written like a linear combination of inferior powers of A.

It is on the basis of this observation that we have led the empirical investigations of our study. Also, to obtain correct estimators of the models parameters and to value forecasts, we have written a set of Pascal programs, trying to respect to the best the rule of good programming and above all, trying to select algorithms with good numerical performances. Thus, concerning the stages of matrix calculations (inversion, singular value decomposition, Schur form, ...) subjacent to the estimation of state space models, we have systematically chosen the algorithms which have the properties of numerical stability. It is in particular the Gauss algorithm with full pivoting for matrix inversion, the Golub and Reinsh (see Forsythe et al (1977)) for the decomposition into singular elements and the Willkinson algorithm based on QR iterations with double implicit translation for the factorization in Schur form.

For the validation of a program, it is usual to carry out sets of tests which permit to compare the results suupplied by this program with some well known solutions. Also, on the basis of the empirical tests that we have executed, for the procedures of matrix numerical calculation previously mentioned, as well as for the ones associated to different stages of the state space method, we could conclude to the validity of our programs.

2. Applications to some caribbean variables

A national economy is too complex for us to pretend to describe it precisely, let alone try to forecast the evolution af all the data and phenomena which characterize it. Thus, models and previsions are usually elaborated for key variables representative of economic activity. These are the main aggregates of the real sphere of the economic activity, the main prices and the main monetary and financial variables.

In industrialised countries, this task is carried out by various public or private organisms which regularly publish general results targetting a large audience and specific results required by government officials who use them.

As far as the Caribbean is concerned, an inventory of fixtures of this practice lead to a mitigated statement. It is true that a lot of efforts have been made since the 70s and have resulted in the awareness that it was necessary to build operational models that could enlighten economic development policy. However, important obstacles persist and slow down this process. One just has to ask oneself the two following questions to illustrate this state of fact. With a potential of about ten models for the countries having the greatest modeling experience, that is to say Jamaica, Barbados, Guyana and Trinidad and Tobago, how many of these models have actually been used for any evaluation of economic measure ? Apart from the private sector and that of banks in particular, can it be asserted that caribbean decision-makers commit themselves to the development of studies for preparation and evaluation of economic policy ?

Therefore, in ministries as well as in big companies, Caribbean people in charge economically have and will have to take important decisions concerning large amounts of money. As one cannot rule out uncertainty, econometric studies bearing on forecasts and simulation, completed if need be by expert opinion, constitute for sure a solid support to help decision-making.

For example, with an industry which now makes up about 50% of the export volume and nearly 40% of State resources, commercializing trinidadian oil implies follow-up and forecasting of oil prices on the international market. As for Guyana's bauxite it is necessary to regulate production and sell at the best prices. In the same way, revenues generated by tourism depend on the international overall economic situation and exchange rate. They are of course of dramatic importance to most caribbean countries and consequently, forecasting variables that describe the number, the length of stay and the expenses of the tourist is of paramount importance.

For these two examples alone, a quantitative prospective analysis on the basis of a conventional structural econometric model turns out to be a tricky exercice since quarterly or monthly time series of the most significant explanatory variables are not available. More generally it is suitable to point out that time series of numerous caribbean economic variables are missing totally or have missing observations, are too short or do not have the right periodicity (Watson (1995)). Faced with this situation a question naturally crops up : how to deal with such a statistical lack ?

In another article published in 1995, Watson also raised this thorny question in the field of the estimation of a cointegrated simultaneous equation model. We share his point of view concerning the adoption of a pragmatic approach which consists in using a method that works. In this respect it seems logical to seek a solution to the problem of the elaboration of economic forecasts on the side of time series methods.

Applications for the forecasting of caribbean macroeconomic variables have already been made by many authors. To name only one, Greenidge (1995) adjusted different models related to exponential smoothing and Box-Jenkins methodology in order to propose forecasts of barbadian money supply.

However, if we refer to econometric literature, it is not too much to say that among the methods of time series approach, VAR models constitute the most interesting tools for forecasting. Concerning their efficiency, numerous works have shown that they yield comparable, if not superior results to those of prevision institutes (Wallis (1989), Doz and Malgrange (1992), Clément and Germain (1993)).

Then, the idea that this conclusion remains valid for the state space models can be put forward immediately[5] if we consider the fact that these last represent a generalisation of VAR models and that one can switch from one to the other easily (Aoki (1987)).

A pertinent way to appreciate empirical behaviour of an econometric method is to perform applications on time series with various configurations : univariate or multivariate; short or long data periods; annual, quarterly or mensual periodicity; stable or volatile data. So, in this second section, we apply the state space method on a the following data :

- the Barbados number of tourists in monthly data from january 1992 to april 1995.
- a quarterly sample of the money (m_t), the index of industrial production (y_t) and the index of consumer prices which spans from the first quarter of 1973 to the third quarter of 1994.
- the Trinidad and Tobago annual vector $(y_t \quad c_t \quad i_t \quad x_t \quad m_t)'$ of

[5] A comparative study aimed at confronting forecasts of the state space procedure and those given by VAR methodology and others methods would be quite useful.

2.1 The tourism example

On the world economy level, as well as on a more restricted level, be it for a country or for a geographical zone, the tourism industry has known a progressive development which has become more pronouced over the past few years with the easing of restrictions on air transport. If we rely on the OMT statistics, the total nomber of tourists arrivals increased from 60 million in 1960 to 284 million en 1980, before reaching 528 million in 1994. In the same way, receipts increased from 7 billion US dollars in 1960 to 103 billion in 1980 and 341 billion in 1994. The examination of international exchanges amount also shows that tourism ranks first among the various categories of exchanged products, far ahead of petrol and automobile industry. For industrialized countries which mainly benefit from the receipts of international tourism, this sector weighs a lot in terms of employment, consumption and investment. For most of the other countries and especially for the caribbean countries, the development of tourism is obviously considered as a strategy of economic development. On this point, we can note that the caribbean countries which have known the highest growth rates over the past few years are those which opted for a policy of development which gives a major role to tourism.

These brief observations are greatly enough to underline, if it was necessary, the interest of studies aiming at measuring the macroeconomic impact of tourists arrivals in a country and the necessity of forecasting in this field. Concerning the forecasts horizon, this interest stands at many levels for, as Witt and Witt (1995) mentioned it, "short term forecasts are required for scheduling and staffing, medium term forecasts for planning tour operator brochures and long term forecasts for investment in aircraft, hotels and infrastructure".

Concerning the stricto sensu forecast, the work relating to the tourism demand, are relatively abundant in literature. It started during the good period of macroeconometric modelling stemming from the keynesian inspiration, and which concerned principally the specification of behaviour relations aiming at explain the determiners of tourism demand. In the concern to improve the forecasts precision, applications have taken place, afterwards, with various methods for time series analysis. For example, González et Moral (1995) have suggested the use of a model of decomposition including an indicator of revenue, two prices indexes, a stochastic trend and a stochastic seasonal component in order to explain the tourism external demand for Spain. For a more overall vision on these studies we can refer to the literature review of Witt and Witt (1995).

In the particular case of Caribbean countries, the least we can say is that these studies are still in few number. Among the contributions on that matter,we can name Rosensweig (1988) which has tested several specifications so as to analyse the prices-effects on the tourist results of some caribbean countries. More recently, Whitehall et Greenidge (1996) have tested cointegrated relations to examine the problem of tourism maturity fo Barbados and Bermuda.

As an indicator of the tourism activity, we have retained the monthly series of the number of tourists who visited Barbados on the period going from january 1986 to july 1995. Of course, the indicators measuring the tourism activity of a country are usually affected by seasonal variations. For this Barbadian series we know that its seasonality can be describe by a deterministic seasonal pattern (Maurin(1995)). However, we kept the unadjusted data and opted for applying the method on centred data.

With five lags, the 5×5 Hankel matrix for the number of tourists has the following singular values :

$$\sigma_i : \quad 23.7710 \qquad 19.2921 \qquad 14.49998 \qquad 2.4334 \qquad 0.1381$$

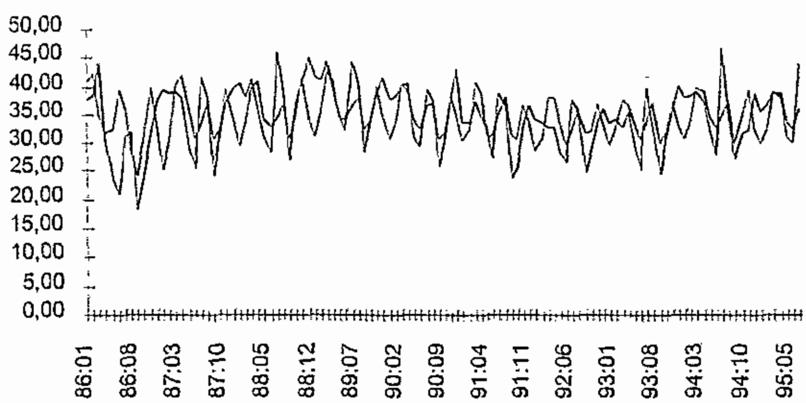
There is a gap from the third value to the fourth value which suggest to retain 3 state variables to adequately synthetize the dynamic evolution of y_t. Results of our Pascal program are reported in table 1. They show that the estimators properties in the particular case of a scalar series are well respected. The figure 1 illustrating comparison of actual and in-sample forecast values suggest the good abilities of the model, in view of the great fluctuations in the observed number of tourists.

Table 1 : Estimation results with $\hat{n} = 3$

			coefficients			
	$\hat{A}$		$\hat{G}$		$\hat{C}$	
-0.2313	0.8876	0.0895	-0.0393	-3.2395	-1.2974	-1.9194
-0.8876	0.1571	0.4531	0.0863			
0.0895	-0.4531	0.6909	-0.0736			
			covariances			
	$\hat{\Pi}$		$\hat{\Delta}_e$		$\hat{\Delta}_0$	
0.7010			75.0302	35.5278		
0.1671	0.5908					
0.0259	0.0861	0.3647				
			summary statistics[6]			
AVERAGE	MAD	FPE	RMSE			
-0.0447	4.4382	29.8202	5.4608			

Time plots of actual and in-sample forecast values

figure 1. number of tourists



[6] AVERAGE is the average error, MAD is the mean absolute deviation error, FPE is the final prediction error, and RMSE is the root mean squared error.

2.2. The Barbados money and output data

Specification of models aiming to analyse the relationships between money and output in Barbados can lead immediatly to think about the controversial questions on realist and monetarist interpretrations of business cycle in this country. As an alternative empirical strategy to shed a ligth on the explanations of the money-output correlation, state space models will be probably very interesting. But here, we focus our interest only on the question of forecasting these variables by taking into account of feedback among them.

We have conducted our estimation exercice on the basis of 88 quarterly observations, from 1973 to 1994. Chosing 4 lags we have calculated a 8×8 Hankel matrix whose estimated singular values are :

4.5333	0.0495
1.2591	0.0071
0.1431	0.0018
0.0729	0.0004

Setting $\hat{n} = 2$, the coefficients A, G and C can be estimated as well as the initial condition by the backcasting technique. These estimates are presented in table 2.

With the initial condition $\hat{x}_0 = \begin{pmatrix} 1.8236 \\ 0.7487 \end{pmatrix}$ we have used the estimated model to obtain in-sample forecast for the money and output series. Summary statistics of these in-sample forecasts are also reported in table 2.

Table 2 : Estimation results for money and output

		coefficients			
$\hat{A}$		$\hat{G}$		$\hat{C}$	
0.9412	-0.0121	-0.7655	-0.3321	-0.9386	0.3245
-0.0916	0.8924	0.7835	-1.0041	-0.6818	-0.5570
		covariances			
$\hat{\Pi}$		$\hat{\Delta}_e$		$\hat{\Delta}_0$	
0.9528		1.0394		0.9886	
0.0130	0.8839	0.5186	1.1452	0.4959	0.9886
		summary statistics			
	Monney	IPI			
AVERAGE	-0.0009	0.0078			
MAD	0.1411	0.5519			
FPE	0.0199	0.3046			
RMSE	0.1069	0.4406			

Time plots of actual and in-sample forecast values

figure 2. money

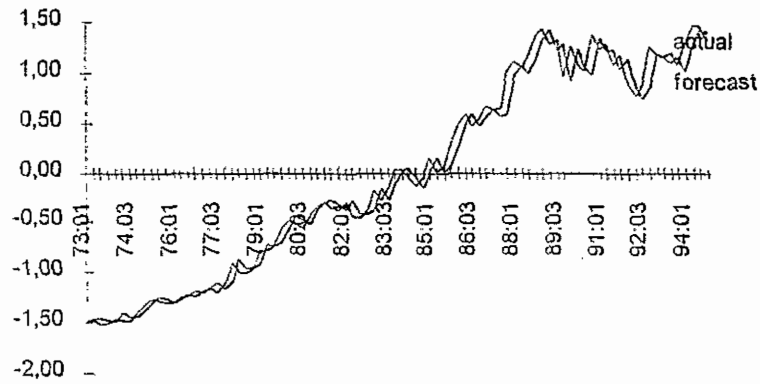
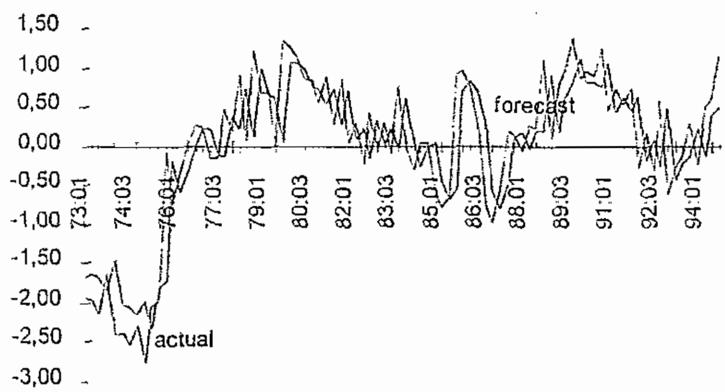


figure 3. index of industrial production



2.3. The example of Trinidad and Tobago's macroeconomic aggregates

The studied series here gathers annual macroeconomic aggregates of the GDP demand identity. The y_t vector is then constitute of the following variables : national output (GDP), total consumption expenditure (FCE), gross capital formation (GCF), imports of goods and non factor services (M) and exports of goods and non factor services (X). The data are relative to the period 1970-1994 and correspond to constant prices values of 1987 prices.

We estimated the model after subtracting the means. We have retain two lags, which is an acceptable value for annual data. The singular value decomposition has thus been applied on a 10×10 Hankel matrix ans has led to the following ratios (to the first singular value):

1	0.446	0.065	0.011	5.02E-3
3.54E-3	2.25E-3	1.19E-3	4.53E-4	5.62E-5

Based on this we chose $\hat{n} = 2$. Table 3 summarizes the estimation results, statistics on in-sample forecasts and error autocorrelations.
These results with figure 4 to 8 bring out that the procedure performs well on actual data.

Table 3 : Estimation results for the vector $(GDP_t \quad FCE_t \quad GCF_t \quad M_t \quad X_t)'$

coefficients

$\hat{A}$		$\hat{G}$					$\hat{C}'$				
0.8944	0.0528	-0.3490	0.2418	0.1077	0.0213	-0.1585	-0.7356	-0.9213	-0.2833	-0.5429	-5.7086
0.1435	0.8303	0.4778	-0.3643	0.0515	0.3657	-0.0664	2.322.3	2.5526	1.1157	1.5669	-0.6149

covariances

$\hat{\Pi}$		$\hat{\Delta}_e$					$\hat{\Delta}_0$				
0.9579		7.5898					7.4702				
-0.0219	1.0169	7.2891	8.0515				7.1764	7.7065			
		4.1227	3.4334	3.7803			4.0536	3.2772	3.8032		
		4.3980	4.8484	2.2921	3.1697		4.3250	4.6337	2.3407	3.0649	
		3.3117	3.7862	1.6882	2.4912	49.431	4.3250	4.6337	2.3407	3.0649	35.2174

summary statistics

	GDP	FCE	GCF	M	X
AVERAGE	0.0325	0.0170	-0.0660	-0.0218	-0.0891
MAD	0.5628	0.8943	1.1212	0.5369	0.7232
FPE	0.5070	1.0805	1.7055	0.5079	0.9390
RMSE	0.7123	1.0395	1.3060	0.7127	0.9690

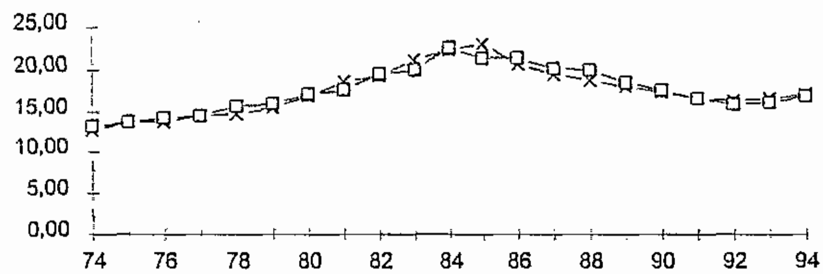
error autocorrelations, lag 1-5

	lag 1	lag 2	lag 3	lag 4	lag 5
GDP	0.1862	-0.0570	0.0518	-0.2323	-0.1448
FCE	0.4825	0.3358	0.2688	0.0153	-0.0295
GCF	-0.0121	0.0242	0.3306	0.0249	-0.0638
M	-0.1111	-0.3156	0.1396	-0.0083	-0.0100
X	-0.0267	-0.0978	0.0234	-0.0182	-0.0385

Time plots of actual and in-sample forecast values

× : observed values : forecasts

figure 4. GDP



Time plots of actual and in-sample forecast values

× : observed values   : forecasts

figure 5. Consumpion

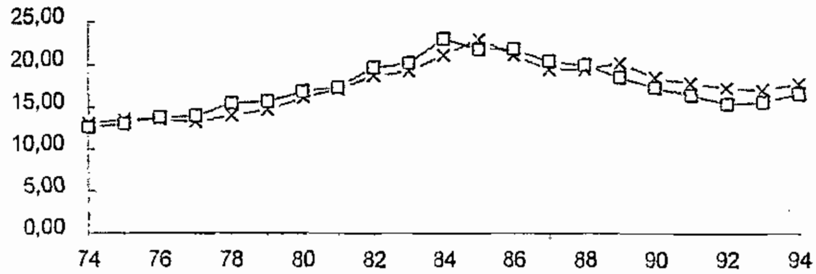


figure 6. gross capital formation

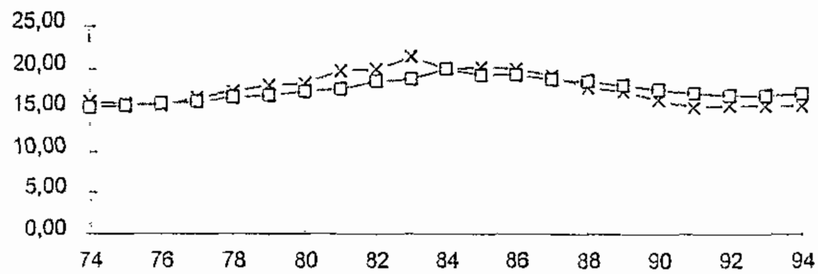


figure 7. imports

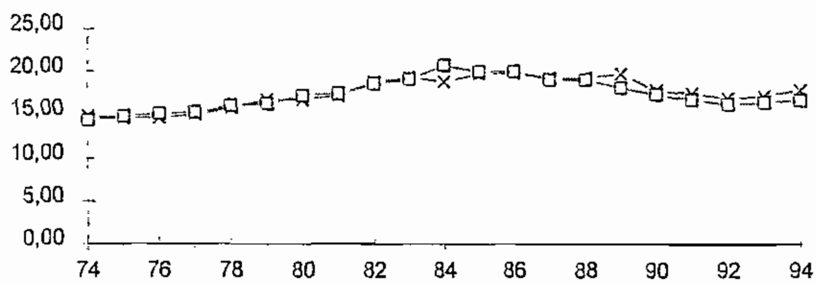
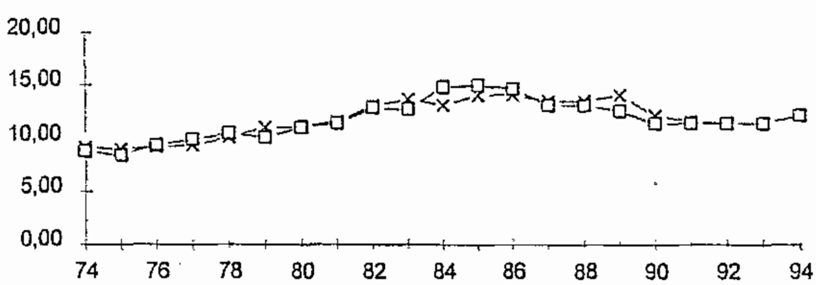


figure 8. exports



Conclusion

Since fifteen years or so, economists have endlessly sought models and concepts to describe the best they can evolutions of economic variables. Aiming at making up for the deficiencies of traditional macroeconometric models, this research has brought new tools which have considerably enriched and renewed the econometric analysis (causality, non stationarity, cointegration, seasonality, non linearity, ...).

The object of this article was to present the modelling process by state space variables and also to try to demonstrate that this approach constitutes a satisfactory alternative compared to other forecasting methods.

We especially took an interest in the computer implementation making sure that the algorithms that we chose were acknoledged for their performances as regards numerical precision. The illustrations that we made have thus shown that state space models provide good results.

In the particular case of Caribbean countries, this study allowed us to show that the state space models can constitute a very interesting complementary tools for the forecaster.

Broadly speaking, considering their low estimation cost and their great flexibility to modelize data of various types, we think that these models will definitely not be out of place in the "toolbox" of any forecasting institute.

Appendix 1
Case of a scalar series

When the series is univariate, we can immediately bring to the fore some characteristics of the estimated parameters.
First, Hankel matrix is symmetrical, its singular values decomposition implies thus $U = \Sigma V$ with $\Sigma^2 = I$.

Consequently, C and Ω are defined in such a way that

$$C' = \Sigma\Omega$$

For the matrix A, one can prove that each coefficient a_{ij} verifies the equality $a_{ij} = \pm a_{ij}$.

Lastly, if $n = 1$, the Riccati equation amounts to a quadratic equation :

$$c^2\pi^2 - \left[\lambda_0(1-a^2) + 2\omega ac\right]\pi + \omega^2 = 0$$

Appendix 2 :
Mittnik procedure

The original hypothesis is that the output $\{y_t\}$ are generated by a AR process. In this case, the Hankel matrix is built from least squares estimators of the parameters of the AR process.

The Hankel matrix

y_t verifies the relation

$$y_t = \sum_{i=1}^{p} M_i y_{t-i} + e_t$$

where $\{e_t\}$ is a mean zero noise process, serially uncorrelated with covariance matrix Σ and the M_i are matrices with dimension q x q.

Let $\underset{kq,q}{\Theta} = \begin{bmatrix} M_1 & M_2 & \cdots & M_k \end{bmatrix}^t$ be the block matrix of the matrices of parameters of the AR model.

The least squares estimator of the $\hat{\Theta}$ is :

$$\hat{\Theta} = (X'X)^{-1}X'Y$$

where : $\underset{(N-k),q}{Y} = \left[y_{k+1} \cdots y_N\right]^t$

$$\underset{kxq,N-k}{X}{}^{t} = \begin{bmatrix} y_k & y_{k+1} & \cdots & y_{N-1} \\ y_{k-1} & y_k & \cdots & y_{N-2} \\ . & . & \cdots & . \\ . & . & \cdots & . \\ . & . & \cdots & . \\ y_1 & y_2 & \cdots & y_{N-k} \end{bmatrix}$$

k is an integer which must be carefully chosen under the constraint $k \geq p$. Indeed, the size of the model depends on k.

The hypermatrix $\hat{H}$ is defined by :

$$\underset{k\times q,k\times q}{\hat{H}}{}^{t} = \begin{bmatrix} \hat{M}_1 & \hat{M}_2 & \hat{M}_3 & \cdots & \hat{M}_k \\ M & M & \cdots & \hat{M}_k & 0 \\ . & . & . & \cdots & . \\ . & . & . & \cdots & . \\ . & . & . & \cdots & . \\ \hat{M}_k & 0 & 0 & \cdots & 0 \end{bmatrix}$$

Parameters estimation

Here, the decompostion of the matrix $\hat{H}_k$ is

$$\hat{H}_k = \hat{M}O \times \hat{M}C$$

MO *et* MC stands respectively for the observability and commandability matrices of a modified system specified as follows :

$$z_{t+1} = Fz_t + Gy_t$$

$$y_t = Cz_t + w_t$$

For this system, we have : $M_i = CF^{i-1}G$. It follows from that :

$$\underset{k\times q,k\times q}{MO} = \begin{bmatrix} C & CF & \ldots & CF^{k-1} \end{bmatrix} \text{ and } \underset{k\times q,k\times q}{MC} = \begin{bmatrix} C & FG & \ldots & F^{k-1}G \end{bmatrix}$$

As previously we perform the singular value decomposition of the matrix $\hat{H}_k$. MO and MC are then defined by :

$$MO = \hat{U}\hat{\Sigma}^{1/2} \text{ and } MC = \hat{\Sigma}^{1/2}\hat{V}^t$$

Like the Aoki method, we obtain a model of minimum size by removing the kq-n last colums of Σ. We then obtain an approximation of the Hankel matrix : $\hat{H} = \hat{U}\hat{\Sigma}\hat{V}^t$.

It follows from that new matrices for MO *and* MC :

$$\tilde{O} = \tilde{U}\tilde{\Sigma}^{1/2} \text{ and } \tilde{C} = \tilde{\Sigma}^{1/2}\tilde{V}^t$$

We then deduce the estimators of (A, G, C) in the following way :

$\tilde{C}$ is given by the first q rows of O ;

$\tilde{G}$ is defined by the first q columns of $\tilde{C}$;

A is estimated by $\tilde{A} = \tilde{\Sigma}^{-1/2}\tilde{U}^t\bar{\tilde{H}}\tilde{V}\tilde{\Sigma}^{-1/2} + \tilde{G}\tilde{C}$

$\tilde{H}^{\leftarrow}$ is defined by the transformation $\tilde{H}^{\leftarrow} = \begin{bmatrix} \tilde{H}_2 \\ 0 \end{bmatrix}$ with $\tilde{H} = \begin{bmatrix} \tilde{H}_1 \\ \tilde{H}_2 \end{bmatrix}$

$\tilde{H}_1$ being composed of the first q rows of $\tilde{H}$

This procedure also allows the estimation of the initial state :

$$\hat{z}_0 = (MO^t \times MO)^{-1} MO^t Y^*$$

where Y^* is a column vector defined by

$$\underset{kq \times 1}{Y^*} = \begin{bmatrix} y_1^t & \dots & y_{kp}^t \end{bmatrix}$$

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